## Physics 56400 Assignment \#6 - Due November 8 ${ }^{\text {th }}$

1. The general expressions for $c_{V}^{f}$ and $c_{A}^{f}$ are

$$
\begin{aligned}
& c_{V}^{f}=I_{3}^{f}-2 Q_{f} \sin ^{2} \theta_{W} \\
& c_{A}^{f}=I_{3}^{f}
\end{aligned}
$$

where $I_{3}^{f}$ is the third component of the weak isospin and $Q_{f}$ is the charge of the fermion in units where $Q_{e}=-1$. Complete the entries, both symbolically and numerically, in the following table of couplings for quarks and leptons in the Standard Model using $\sin ^{2} \theta_{W}=0.231$ :

| $\boldsymbol{f}$ | $\boldsymbol{Q}_{\boldsymbol{f}}$ | $\boldsymbol{T}_{\mathbf{3}}^{f}$ | $\boldsymbol{c}_{\boldsymbol{V}}^{f}$ | $\boldsymbol{c}_{\boldsymbol{A}}^{f}$ | $\boldsymbol{c}_{\boldsymbol{L}}^{f}$ | $\boldsymbol{c}_{\boldsymbol{R}}^{f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v_{e}, v_{\mu}, v_{\tau}$ | 0 | $1 / 2$ |  |  |  |  |
| $e^{-}, \mu^{-}, \tau^{-}$ | -1 | $-1 / 2$ |  |  |  |  |
| $u, c, t$ | $2 / 3$ | $1 / 2$ |  |  |  |  |
| $d, s, b$ | $-1 / 3$ | $-1 / 2$ |  |  |  |  |

2. Show that when $\sqrt{s}=M_{Z}$, the total cross section for $e^{+} e^{-} \rightarrow f \bar{f}$, where $f \neq e$, can be written

$$
\sigma_{f \bar{f}}=\frac{12 \pi \Gamma_{e} \Gamma_{f}}{M_{Z}^{2} \Gamma_{Z}^{2}}
$$

where

$$
\Gamma_{f}=N_{c} \frac{G_{F} M_{Z}^{3}}{6 \pi \sqrt{2}}\left(\left(c_{V}^{f}\right)^{2}+\left(c_{A}^{f}\right)^{2}\right)
$$

Calculate the peak cross section for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}, e^{+} e^{-} \rightarrow b \bar{b}$, and $e^{+} e^{-} \rightarrow c \bar{c}$. Also, explain why this expression is not expected to be valid when $f=e$.
3. Prepare three graphs showing $A_{F B}$ as a function of $\sqrt{s}$, for $50<\sqrt{s}<200 \mathrm{GeV}$, for the following processes:

$$
\begin{gathered}
e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \\
e^{+} e^{-} \rightarrow b \bar{b} \\
e^{+} e^{-} \rightarrow c \bar{c}
\end{gathered}
$$

assuming three values for $\sin ^{2} \theta_{W}: 0.22,0.231$, and 0.24 . Which process is most sensitive to variations in $\sin ^{2} \theta_{W}$ ?

