Physics 56400 Assignment #4 – Due October 2nd

1. The Bethe equation for energy loss of a particle with mass M and charge z is written

$$\langle -\frac{dE}{dx} \rangle = \ Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \log(\frac{2m_e c^2 \beta^2 \gamma^2 W_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

in which

$$K = 4\pi N_A r_e^2 m_e c^2 = 0.307075 \text{ MeV} \cdot \text{mol}^{-1} \cdot \text{cm}^2$$

$$W_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + \frac{2\gamma m_e}{M} + \left(\frac{m_e}{M}\right)^2}$$

$$\frac{\delta(\beta\gamma)}{2} = \log\left(\frac{\hbar\omega_p}{I}\right) + \log(\beta\gamma) - \frac{1}{2}$$

$$\hbar\omega_p = \sqrt{\rho\langle Z/A \rangle} \times 28.816\ eV$$

This allows the energy loss per unit length to be expressed in terms of the properties of the material in which it travels, namely the atomic number Z, the atomic mass A, density ρ and the mean excitation energy I. Lists of $\rho, Z/A$, and I for various elements and compounds can be found at

https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients

and the tables referenced therein.

The ionization deposited by protons in various materials as a function of depth is calculated by the program

http://www.physics.purdue.edu/~mjones/phys56400/dEdx.C

which you should examine closely to make sure you understand how it works. Basically, it uses the Bethe equation to calculate energy loss in small steps, and updates the kinetic energy of the proton after each step. The energy losses are simulated in several different materials with their properties obtained from the NIST tables.

Modify the program to simulate the treatment of a brain tumor with a proton beam. The human head should be modeled as a volume with a total diameter of 20 cm containing a brain inside a skull made of bone with a thickness of 7.1 mm.

If the tumor is approximately 5 cm from the surface of the head, and the proton beam has an initial kinetic energy of 350 MeV, what thickness of acrylic absorber is needed to cause the greatest ionization dose to occur at the location of the tumor? Produce some well-documented plots to support your answer.

2. Calcium-48 is an isotope of calcium that can undergo double-beta decay:

$$^{48}Ca \rightarrow {}^{48}Ti + 2\beta^- + 2\bar{\nu}_e$$

and it is speculated that it might undergo neutrinoless double beta-decay:

$$^{48}Ca \rightarrow {}^{48}Ti + 2\beta^- + 0\bar{\nu}_e$$

in which each electron would have a kinetic energy of approximately 2.137 MeV. This decay has been searched for by dissolving a salt of ^{48}Ca in water and looking for the Cherenkov light that would be produced by two mono-energetic electrons. Recall that the number of photons emitted per unit path length, per unit wavelength, is

$$\frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2}\right)$$

a) *Modify the dEdx.C program to calculate the range in water of electrons with this energy.*
b) *Estimate the total number of Chereknov photons emitted along the path in the range of wavelengths from 400 to 800 nm*, assuming that the index of refraction of the ^{48}Ca solution is a constant, $n = 1.33$.
c) *Describe ways that this signature could be distinguished from the normal doublel-beta decay process, which in addition to the two electrons, has two neutrinos in the final state that share the available energy released in the decay.*