Assignment #5

First we need to calculate the time integrated B^0 mixing probability. The probability of observing a B^0 decaying in an unmixed state at time t is

$$P(B^0, t \mid B^0) = \frac{\Gamma e^{-\Gamma t}}{2}\left(1 + \cos \Delta m t\right)$$

and the time integrated probability is

$$P(B^0 \mid B^0) = P_{unmixed} = \frac{\Gamma}{2}\int_0^\infty e^{-\Gamma t}\left(1 + \cos \Delta m t\right) dt$$

$$= -\frac{e^{-\Gamma t}}{2}\Big|_0^\infty + \frac{\Gamma}{2}\int_0^\infty e^{-\Gamma t} \cos \Delta m t \, dt$$

$$= \frac{1}{2} + \frac{\Gamma}{2}\left(\frac{e^{-\Gamma t}\left(\Delta m \sin \Delta m t - \Gamma \cos \Delta m t\right)}{\Delta m^2 + \Gamma^2}\right)\Bigg|_0^\infty$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{\Gamma^2}{\Delta m^2 + \Gamma^2}\right)$$

For the B^0 meson, $\Delta m = 0.506 \times 10^{12}\ \hbar s^{-1}$

$\tau_{B^0} = 1.519 \times 10^{-12}\ s$

so $\Gamma = 0.658 \times 10^{12}\ \hbar s^{-1}$

Hence, $P(B^0 \mid B^0) = 0.814$

and

$P(\bar{B}^0 \mid B^0) = 0.186 = \chi_d$.

There are then 16 possible final states:

Final State	Fraction	$P(\ell^+\ell^-)$	
$B^+\ B^-$	$(f_{B^+})^2 = 0.1156$	1	0.3000
$B^+,\ \overline{B^0}$	$f_{B^+} f_{B^0} = 0.1156$	$1-\chi_d = 0.814$	
$B^+,\ \overline{B_s^0}$	$f_{B^+} f_{B_s^0} = 0.0374$	$1/2$	
$B^+,\ \overline{\Lambda_b}$	$f_{B^+} f_{\Lambda_b} = 0.0714$	1	
$B^0\ B^-$	$f_{B^0} f_{B^+} = 0.1156$	$1-\chi_d = 0.814$	0.2515
$B^0,\ \overline{B^0}$	$(f_{B^0})^2 = 0.1156$	$(1-\chi_d)^2 + \chi_d^2 = 0.697$	
$B^0,\ \overline{B_s^0}$	$f_{B^0} f_{B_s^0} = 0.0374$	$1/2$	
$B^0,\ \overline{\Lambda_b}$	$f_{B^0} f_{\Lambda_b} = 0.0714$	$1-\chi_d = 0.814$	
$B_s^0,\ B^-$	$f_{B_s^0} f_{B^+} = 0.0374$	$1/2$	0.0550
$B_s^0,\ \overline{B^0}$	$f_{B_s^0} f_{B^0} = 0.0374$	$1/2$	
$B_s^0,\ \overline{B_s^0}$	$(f_{B_s})^2 = 0.0121$	$1/2$	
$B_s^0,\ \overline{\Lambda_b}$	$f_{B_s} f_{\Lambda_b} = 0.0231$	$1/2$	
$\Lambda_b\ B^-$	$f_{\Lambda_b} f_{B^+} = 0.0714$	1	0.1852
$\Lambda_b\ \overline{B^0}$	$f_{\Lambda_b} f_{B^0} = 0.0714$	$1-\chi_d = 0.814$	
$\Lambda_b\ \overline{B_s^0}$	$f_{\Lambda_b} f_{B_s^0} = 0.0231$	$1/2$	
$\Lambda_b\ \overline{\Lambda_b}$	$(f_{\Lambda_b})^2 = 0.0441$	1	

The total probability of observing opposite sign lepton pairs is then,

$$P(\ell^-\ell^+) + P(\ell^+\ell^-) = 0.7917$$

and consequently, $P(\ell^+\ell^+) + P(\ell^-\ell^-) = 0.2083$

The expected ratio is $\dfrac{P(\ell^+\ell^+)+P(\ell^-\ell^-)}{P(\ell^+\ell^-)+P(\ell^-\ell^+)} = 3.8$