Physics 56400 Assignment #3

1. The radiation length of a mixture of materials is the mass-weighted sum of the radiation lengths of the pure components:

$$1/X_0^{eff} = \sum w_j / X_0^j$$

For equal molar fractions of Argon and Ethane, the mass fractions are

$$w_{Ar} = \frac{m_{Ar}}{m_{Ar} + m_{Eth}} = \frac{(39.948\ g/mol)}{(39.948\ g/mol) + (30.07\ g/mol)}$$

$$= 0.571$$

and $w_{Ethane} = 1 - w_{Ar} = 0.429$.

From the PDG table of material properties,

$$X_0^{Ar} = 19.55\ g \cdot cm^{-2}$$
$$X_0^{Eth} = 45.66\ g \cdot cm^{-2}$$

Therefore, the effective radiation length of the mixture is

$$X_0^{ArEth} = \left(0.571 \times (19.55\ g \cdot cm^{-2})^{-1} + 0.429 \times (45.66\ g \cdot cm^{-2})^{-1}\right)^{-1}$$

$$= 25.90\ g \cdot cm^{-2}$$

Likewise, $\lambda_I^{Areth} = \left(0.571 \times (119.7\ g \cdot cm^{-2})^{-1} + 0.429 \times (75.9\ g \cdot cm^{-2})^{-1}\right)^{-1}$

$$= 95.95\ g \cdot cm^{-2}$$

and $(dE/dx)_{min}^{ArEth} = 0.571 \times (1.519\ MeV \cdot g^{-1} \cdot cm^2) + 0.429 \times (2.304\ MeV \cdot g^{-1} \cdot cm^2)$

$$= 1.650\ MeV \cdot g^{-1} \cdot cm^2 .$$

The densities of pure Argon and Ethane are

$$\rho_{Ar} = 1.662 \ g/\ell$$

and

$$\rho_{Eth} = 1.263 \ g/\ell .$$

Therefore, the density of 50% Ar / 50% Ethane would be

$$\rho_{ArEth} = \tfrac{1}{2}(1.662 \ g/\ell) + \tfrac{1}{2}(1.263 \ g/\ell)$$

$$= 1.463 \ g/\ell = 1.463 \times 10^{-3} \ g \cdot cm^{-3} .$$

So, detector A has $n_{X_0} = \dfrac{(200 \ cm)(1.463 \times 10^{-3} \ g \cdot cm^{-3})}{(25.90 \ g \cdot cm^{-2})}$

$$= 0.0113 \text{ radiation lengths}$$

of material.

Likewise, there are $n_{\lambda_I} = \dfrac{(200 \ cm)(1.463 \times 10^{-3} \ g \cdot cm^{-3})}{(95.95 \ g \cdot cm^{-2})}$

$= 0.00305$ nuclear interaction lengths of material.

A MIP will deposit

$$\Delta E = (200 \ cm)(1.650 \ MeV \cdot g^{-1} \cdot cm^{-2})(1.463 \times 10^{-3} \ g \cdot cm^{-3})$$

$$= 0.483 \ MeV$$

as it passes through detector A.

For 16 cm of lead and 4 cm of plastic scintillator, the mass fractions are

$$w_{Pb} = \frac{\rho_{Pb}\,\Delta x_{Pb}}{\rho_{Pb}\,\Delta x_{Pb} + \rho_{PVT}\,\Delta x_{PVT}}$$

$$= \frac{(11.350\ g\cdot cm^{-3})(16\ cm)}{(11.350\ g\cdot cm^{-3})(16\ cm) + (1.03\ g\cdot cm^{-3})(4\ cm)}$$

$$= 0.978$$

$$w_{PVT} = 1 - w_{Pb} = 0.022 \quad .$$

The effective radiation length is

$$X_0^{Pb/PVT} = \left(0.978 \times (6.37\,g\cdot cm^{-2})^{-1} + 0.022 \times (43.90\ g\cdot cm^{-2})^{-1}\right)^{-1}$$

$$= 6.49\ g\cdot cm^{-2}$$

Likewise,

$$\lambda_I^{Pb/PVT} = \left(0.978 \times (199.6\ g\cdot cm^{-2})^{-1} + 0.022 \times (81.3\ g\cdot cm^{-2})^{-1}\right)^{-1}$$

$$= 193.4\ g\cdot cm^{-2}$$

and
$$\left(\frac{dE}{dx}\right)_{min}^{Pb/PVT} = 0.978 \times (1.122\ MeV\cdot g^{-1}\cdot cm^2) + 0.022 \times (1.956\ MeV\cdot g^{-1}\cdot cm^2)$$

$$= 1.140\ MeV\cdot g^{-1}\cdot cm^2$$

and the average density is

$$\rho_{Pb/PVT} = 0.8 \times (11.350\ g\cdot cm^{-3}) + 0.2 \times (1.03\ g\cdot cm^{-3})$$

$$= 9.286\ g\cdot cm^{-3} \quad .$$

The number of radiation lengths of material in detector B is then

$$n_{X_0} = \frac{(20\ \mathrm{cm})(9.286\ \mathrm{g\cdot cm^{-3}})}{(6.49\ \mathrm{g\cdot cm^{-2}})} = 28.6$$

The number of nuclear interaction lengths is

$$n_{\lambda_I} = \frac{(20\ \mathrm{cm})(9.286\ \mathrm{g\cdot cm^{-3}})}{(193.4\ \mathrm{g\cdot cm^{-2}})} = 0.960$$

and the energy deposited by a MIP is

$$\Delta E = (20\ \mathrm{cm})(1.140\ \mathrm{MeV\cdot g^{-1}\cdot cm^{2}})(9.286\ \mathrm{g\cdot cm^{-3}})$$

$$= 212\ \mathrm{MeV}$$

For the 180 cm of iron and 20 cm of scintillator, the mass fractions are

$$w_{Fe} = \frac{\rho_{Fe}\Delta x_{Fe}}{\rho_{Fe}\Delta x_{Fe} + \rho_{PVT}\Delta x_{PVT}}$$

$$= \frac{(7.874\ g\cdot cm^{-3})(180\ cm)}{(7.874\ g\cdot cm^{-3})(180\ cm) + (1.03\ g\cdot cm^{-3})(20\ cm)}$$

$$= 0.986$$

$$w_{PVT} = 1 - w_{Fe} = 0.014$$

The effective radiation length is

$$X_0^{Fe/PVT} = \left(0.986 \times (13.84\ g\cdot cm^{-2})^{-1} + 0.014 \times (43.90\ g\cdot cm^{-2})^{-1}\right)^{-1}$$

$$= 13.97\ g\cdot cm^{-2}$$

Likewise,

$$\lambda_I^{Fe/PVT} = \left(0.986 \times (132.1\ g\cdot cm^{-2})^{-1} + 0.014 \times (81.3\ g\cdot cm^{-2})^{-1}\right)^{-1}$$

$$= 131.0\ g\cdot cm^{-2}$$

and $\left(\frac{dE}{dx}\right)_{min}^{Fe/PVT} = 0.986 \times (1.451\ MeV\cdot g^{-1}\cdot cm^2)$

$$+ 0.014 \times (1.956\ MeV\cdot g^{-1}\cdot cm^2)$$

$$= 1.458\ MeV\cdot g^{-1}\cdot cm^2$$

and the average density is

$$\rho_{Fe/PVT} = \frac{(180\ cm)(7.874\ g\cdot cm^{-3}) + (20\ cm)(1.03\ g\cdot cm^{-3})}{180\ cm + 20\ cm}$$

$$= 7.190\ g\cdot cm^{-3}.$$

The number of radiation lengths of material in detector C is

$$n_{X_0} = \frac{(200\text{ cm})(7.190\text{ g}\cdot\text{cm}^{-3})}{13.97\text{ g}\cdot\text{cm}^{-2}} = 103$$

The number of nuclear interaction lengths is

$$n_{\lambda_I} = \frac{(200\text{ cm})(7.190\text{ g}\cdot\text{cm}^{-3})}{131.0\text{ g}\cdot\text{cm}^{-2}} = 11.0$$

and the average energy deposited by a MIP is

$$\Delta E = (200\text{ cm})(1.458\text{ MeV}\cdot\text{g}^{-1}\cdot\text{cm}^2)(7.190\text{ g}\cdot\text{cm}^{-3})$$

$$= 2096\text{ MeV} .$$

Finally, for 10 cm of plastic scintillator,

$$X_0 = 43.90\text{ g}\cdot\text{cm}^{-2}$$

$$\lambda_I = 81.3\text{ g}\cdot\text{cm}^{-2}$$

$$\left(\frac{dE}{dx}\right)_{min} = 1.956\text{ MeV}\cdot\text{g}^{-1}\cdot\text{cm}^2$$

Thus,

$$n_{X_0} = \frac{(10\text{ cm})(1.03\text{ g}\cdot\text{cm}^{-3})}{43.90\text{ g}\cdot\text{cm}^{-2}} = 0.235$$

$$n_{\lambda_I} = \frac{(10\text{ cm})(1.03\text{ g}\cdot\text{cm}^{-3})}{81.3\text{ g}\cdot\text{cm}^{-2}} = 0.127$$

$$\Delta E = (10\text{ cm})(1.03\text{ g}\cdot\text{cm}^{-3})(1.956\text{ MeV}\cdot\text{g}^{-1}\cdot\text{cm}^2)$$

$$= 20.1\text{ MeV}$$

2. The probability that an electromagnetic particle travels a distance x through a material with radiation length X_0, without interacting is

$$P(x) = e^{-x/X_0}$$

Likewise, the probability that a hadronic particle travels a distance x through a material with nuclear interaction length λ_I without interacting is

$$P(x) = e^{-x/\lambda_I}.$$

For an electron/positron/photon, the probability of traveling through section A without interacting is

$$P_A = e^{-0.0113} = 0.989$$

The probability that it would travel through section B without interacting is

$$P_B = e^{-28.6} = 0$$

Likewise, there is no chance that an electron/positron/photon would ever reach sections C or D.

The probabilities that a proton/neutron would travel through the different sections without interacting are

$$P_A = e^{-0.00305} = 0.997$$
$$P_B = e^{-0.96} = 0.383$$
$$P_C = e^{-(11.0+0.96)} = 6.4\times 10^{-6}$$
$$P_D = e^{-(11.0+0.96+0.13)} = 5.6\times 10^{-6}.$$

Because muons only interact by ionization, we do not expect it to interact in any section of the detector, so

$$P_A = P_B = P_C = P_D = 1.$$

In practice, low energy muons would lose sufficient energy to be stopped in the detector. Certainly, they would need to have more than about 2500 MeV to pass through all parts of the detector.

3. Electrons, positrons and protons would deposit

$$\Delta E_A = 0.483 \text{ MeV}$$

in section A. The probability of interacting in section B is very high, so we expect that

$$\Delta E_B = 50 \text{ GeV}$$

For photons, there is no ionization in section A, so

$$\Delta E_A = 0$$
$$\Delta E_B = 50 \text{ GeV}$$

For protons, $\Delta E_A = 0.483$ MeV
$\Delta E_B = 212$ MeV
but the probability of interacting in section C is high, so $\Delta E_C \approx 50$ GeV.

For neutrons, $\Delta E_A = \Delta E_B = 0$ while $\Delta E_C = 50$ GeV

For muons, $\Delta E_A = 0.483$ MeV, $\Delta E_B = 212$ MeV, $\Delta E_C = 2096$ MeV and $\Delta E_D = 20.1$ MeV.