Physics 56400 Assignment #2

1. A particle with mass M_A and momentum p_A has energy

$$E_A = \sqrt{M_A^2 + p_A^2}$$

and boost parameters $\gamma = E_A / M_A$ and $\gamma\beta = p_A / M_A$.

When it decays into particles with masses m_a and m_b, their energies in the rest frame of particle A are

$$E_a^* = \frac{M_A^2 + m_a^2 - m_b^2}{2M_A}$$

$$E_b^* = \frac{M_A^2 + m_b^2 - m_a^2}{2M_A}$$

If partice a is emitted with polar angles θ^*, φ^* in the A rest-frame, then in the lab frame,

$$p_{az} = \gamma p^* \cos\theta^* + \gamma\beta E_a^*$$

and $p_{bz} = -\gamma p^* \cos\theta^* + \gamma\beta E_b^*$

where $p^* = \sqrt{E_a^{*2} - m_a^2} = \sqrt{E_b^{*2} - m_b^2}$ is the momentum of either particle in the A rest-frame.

Without loss of generality assume that particle a is the lighter of the two final state particles, so that $E_a < E_b$.

If we require that $p_{az} > 0$ then we must always have $\gamma\beta E_a^* > \gamma p^*$ which can also be written

$$p_A E_a^* > E_A p^*$$

Thus, $p_A > E_A p^* / E_a^*$

We need to express this in terms of p_A and M_A. Thus,

$$p_A^2 > E_A^2 \left(\frac{p^*}{E_a^*}\right)^2 = \left(p_A^2 + M_A^2\right)\left(\frac{p^*}{E_a^*}\right)^2$$

Therefore, $$p_A^2\left(1 - \left(\frac{p^*}{E_a^*}\right)^2\right) > M_A^2 \left(\frac{p^*}{E_a^*}\right)^2$$

$$p_A > M_A \frac{\beta_a^*}{\sqrt{1-\beta_a^{*2}}} = M_A \gamma_a^* \beta_a^* =$$

where $\beta_a^* = \frac{p^*}{E_a^*}$ is the velocity of particle a in the A rest frame.

This can also be expressed

$$p_A > M_A \frac{p^*}{m_a}$$

2. In the lab frame, the e^- and e^+ beams have 4-momenta

$$p_{e^-} = \left(E_{e^-}, 0, 0, \sqrt{E_{e^-}^2 - m_e^2} \right)$$

$$p_{e^+} = \left(E_{e^+}, 0, 0, -\sqrt{E_{e^+}^2 - m_e^2} \right)$$

Because $m_e \ll E_{e^\pm}$ we can make the approximation

$$p_{e^-} = (E_{e^-}, 0, 0, E_{e^-})$$

$$p_{e^+} = (E_{e^+}, 0, 0, -E_{e^+})$$

In the center of mass frame, the 4-momentum is

$$p_{cm} = (E_{cm}, 0, 0, 0)$$

while in the lab frame the sum of the beam's 4-momenta is

$$p_{lab} = (E_{e^-} + E_{e^+}, 0, 0, E_{e^-} - E_{e^+})$$

Since $p_{cm}^2 = p_{lab}^2$ is a Lorentz invariant,

$$E_{cm}^2 = (E_{e^+} + E_{e^-})^2 - (E_{e^-} - E_{e^+})^2$$

$$= 4 E_{e^+} E_{e^-}$$

and therefore, $E_{cm} = 2\sqrt{E_{e^+} E_{e^-}}$

In the case of the PEP-II storage ring,

$$E_{e^-} = 9.0 \text{ GeV}$$
$$E_{e^+} = 3.1 \text{ GeV}$$

and therefore $E_{cm} = 10.56$ GeV

The velocity of the center-of-mass frame can be written

$$\beta_{cm} = \frac{p_{cm}}{E_{cm}} = \frac{E_{e^-} - E_{e^+}}{E_{e^-} + E_{e^+}}$$

$$= \frac{(9.0\text{ GeV}) - (3.1\text{ GeV})}{(9.0\text{ GeV}) + (3.1\text{ GeV})} = 0.488 .$$

3. In the center-of-mass frame, both D mesons will have equal and opposite transverse momentum.

Both will have energy $E_D = \frac{E_{cm}}{2}$

and therefore, $|\vec{p}_D| = \sqrt{\left(\frac{E_{cm}}{2}\right)^2 - m_D^2}$

This is the maximum possible p_T with which they could be produced.

$$p_T^{max} = \sqrt{\left(\frac{8\text{ GeV}}{2}\right)^2 - (1.870\text{ GeV})^2}$$

$$= 3.536\text{ GeV} .$$

If the proper lifetime of a D-meson is $\tau_D = 1.04$ ps, then the mean transverse decay length is

$$d = \gamma\beta c\tau_D = \frac{p_T^{max}}{m_D} \cdot c \cdot \tau_D$$

$$= \left(\frac{3.536\text{ GeV}}{1.87\text{ GeV}}\right)(0.2998\text{ mm/ps})(1.04\text{ ps})$$

$$= 0.590\text{ mm} .$$

4. When $p_D = 200$ GeV (it is probably okay to ignore m_D since $m_D \ll E_D$), the mean decay length would be

$$d = \gamma\beta c \tau_D = \left(\frac{200 \text{ GeV}}{1.87 \text{ GeV}}\right)(0.2998 \text{ mm/ps})(1.04 \text{ ps})$$

$$= 33.35 \text{ mm}.$$

For this reason, fixed target experiments at Fermilab provided some of the most precise measurements of D- lifetimes.