

Physics 56400 Assignment # 1

1. An iron nucleus has a radius of $R = 4.73 \text{ fm}$ so the cross section is

$$\begin{aligned} \sigma &= \pi R^2 = \pi (4.73 \times 10^{-15} \text{ m})^2 \\ &= 7.03 \times 10^{-29} \text{ m}^2 \\ &= 7.03 \times 10^{-25} \text{ cm}^2 \\ &= 0.703 \text{ barn.} \end{aligned}$$

The probability that a beam particle will hit an iron nucleus is the ratio of areas subtended by the beam and the cross sectional areas of all iron nuclei in the path of the beam:

$$P = \frac{N_T \sigma}{A}$$

$$\text{where } N_T = N_A \frac{\rho}{m} A \Delta x$$

where ρ and m are the density and atomic mass of iron, and N_A is Avogadro's number.

The rate of interactions is then

$$\begin{aligned} R &= \Phi A \cdot P = \Phi A N_A \frac{\rho}{m} \Delta x \sigma \\ &= \Phi \pi r^2 N_A \frac{\rho}{m} \Delta x \sigma \end{aligned}$$

For iron, $\rho = 7.874 \text{ g}\cdot\text{cm}^{-3}$ and $m = 55.845 \text{ g}\cdot\text{mol}^{-1}$.

Thus, for a circular beam with a radius of $r = 1 \text{ mm}$ and $\Phi = 10^{12} \text{ cm}^{-2} \text{ s}^{-1}$,

$$R = (10^{12} \text{ cm}^{-2} \text{ s}^{-1}) \cdot \pi \cdot (0.1 \text{ cm})^2 (6.022 \times 10^{23} \text{ mol}^{-1}) \cdot \frac{(7.874 \text{ g}\cdot\text{cm}^{-3})}{(55.845 \text{ g}\cdot\text{mol}^{-1})} \times (0.5 \text{ cm}) (7.03 \times 10^{-25} \text{ cm}^2)$$

$$= 9.37 \times 10^8 \text{ s}^{-1}.$$

2. The differential cross section for hard-sphere scattering is

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} R^2$$

where $R = 4.73 \text{ fm}$.

The cross section integrated over $\theta > \pi/2$ is just half the total solid angle.

Thus, this is half the rate of inclusive scattering:

$$R_{\theta > \pi/2} = \frac{1}{2} R = 4.69 \times 10^8 \text{ s}^{-1}.$$