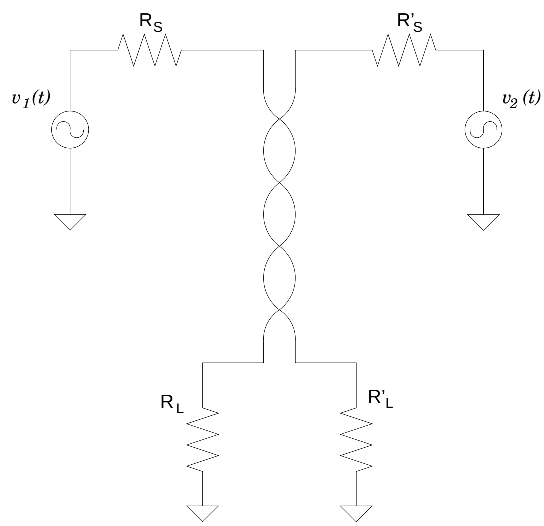
Physics 536 - Assignment #4

1. Digital data is frequently transmitted on cables that consist of pairs of conductors twisted together. If each conductor carries equal and opposite currents, then their radiated electromagnetic fields cancel. However, if there is an imbalance in the current, energy can be radiated and cause interference in other circuits.

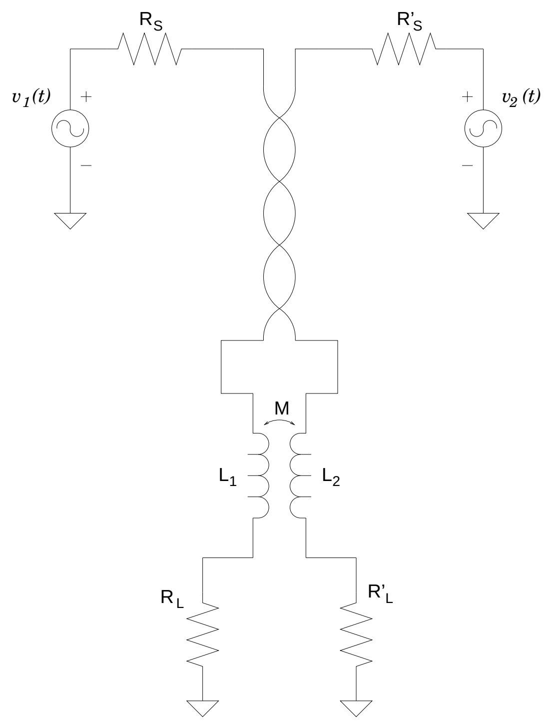
(a) Consider the following model for a twisted pair, driven by two voltage sources, $v_1(t) = V_1 e^{i\omega t}$ and $v_2(t) = V_2 e^{i\omega t}$ as shown:



in which R_S and R'_S represent the impedances of the driving voltage sources and R_L and R'_L represent resistive loads at the receiving end of the cable. Show that, in general, equal and opposite currents will flow in the two conductors only when $V_1 = -V_2$ and $R_S + R_L = R'_S + R'_L$.

Considering that manufacturing tolerances on components are typically only 1-5%, argue that it would be possible, if not likely, that an imbalanced current would flow and become a source of electromagnetic interferance.

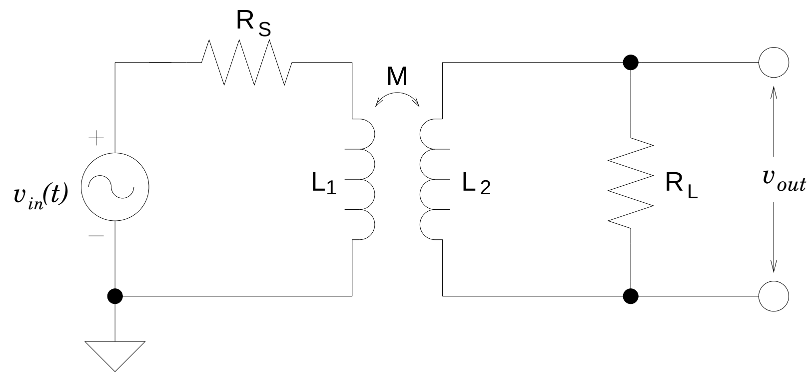
(b) A similar circuit uses a transformer to terminate the twisted pair in the following configuration:



Assuming $L_1 = L_2$ and perfect coupling between the transformer windings, and that $\omega \gg R/L$, show that the currents that flow in the two conductors are now equal and opposite, regardless of whether $V_1 = -V_2$ or whether $R_S + R_L = R'_S + R'_L$.

In this configuration, the transformer is sometimes called a "balun", and it forces equal and opposite currents to flow in the two conductors, minimizing radiated electromagnetic energy. These magnetic components are sometimes integrated into the RJ45 connectors used for Ethernet.

2. This problem investigates the parameters that affect the physical size of transformers. Consider the transformer circuit:

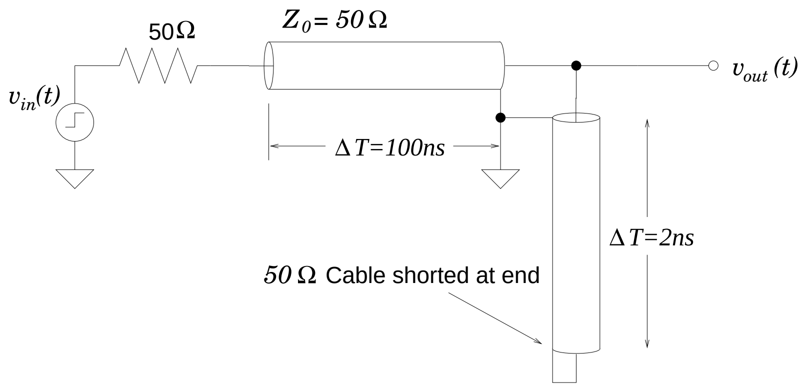


In class, we derived the relation

$$V_{out} = \frac{i\omega M V_{in} R_L}{R_S R_L + i\omega(L_1 R_L + L_2 R_S)} \tag{1}$$

which applies in the case of perfect coupling, $M = \sqrt{L_1 L_2}$.
(a) Calculate the frequency, ω_0, at which the magnitude of the ratio V_{out}/V_{in} is $1/\sqrt{2}$ its value when $\omega \to \infty$.
(b) Suppose a transformer with a turns ratio of 10:1 was used to convert 110 VAC to 11 VAC. If the voltage source had an impedance $R_S = 0.5\ \Omega$ and the load had a resistance of $1\ \Omega$, what primary inductance, L_1, would be required such that $\omega_0 < 190\ \text{s}^{-1}$, which is about half the 60 Hz line frequency?
(c) By arguing that the primary winding can be thought of as an ideal solenoid, show that the physical volume of a transformer is directly proportional to its inductance and therefore inversely proportional to the frequency, ω_0.
(d) Explain why it might be desirable to use an AC voltage source with a frequency of 400 Hz instead of 60 Hz, in such an application. In fact, it is not uncommon to use a motor-generator to produce 400 Hz AC power for industrial applications.

3. Consider the following circuit:



in which $v_{in}(t)$ is an ideal voltage source:

$$v_{in}(t) = \begin{cases} 0 \text{ V} & \text{for } t < 0 \\ 1 \text{ V} & \text{for } t > 0 \end{cases} \tag{2}$$

(a) Sketch, as accurately as possible, the voltage observed at $v_{out}(t)$. This configuration is sometimes used to produce short pulses without the use of fast timing circuits.
(b) At some point the voltage source $v_{in}(t)$ will return return to zero volts. Sketch the voltage observed at $v_{out}(t)$ when this happens.