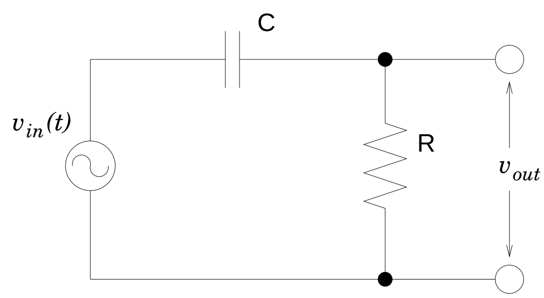
Physics 536 - Assignment #3

1. Consider the high-pass RC filter circuit:

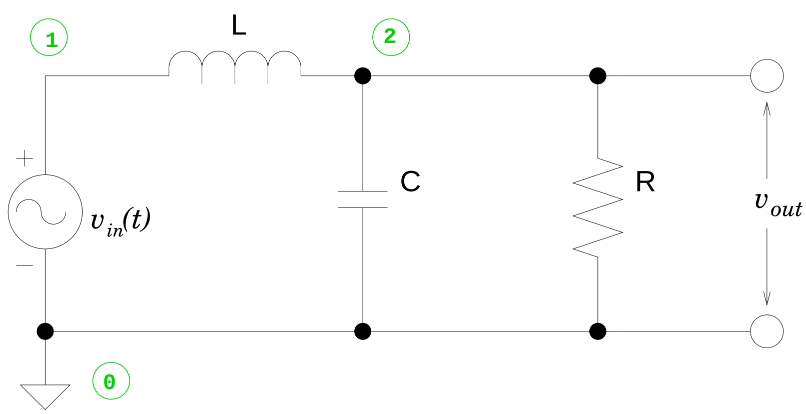


When $v_{in}(t) = V_{in}e^{i\omega t}$, the Thevenin equivalent circuit would consist of an ideal voltage source V_{Th} in series with an impedance Z_{Th}, both of which depend on the frequency, ω.

(a) With no additional load across the resistor, calculate V_{out}. How does V_{out} behave in the low-frequency ($\omega \ll 1/RC$) and high-frequency ($\omega \gg 1/RC$) limits?

(b) Calculate the Thevenin equivalent impedance Z_{Th} for the circuit where the impedance of the resistor is R and the impedance of the capacitor is $i/\omega C$. How does Z_{Th} behave in the low-frequency and high-frequency limits?

2. Consider the following low-pass filter circuit:



(a) Assuming $v_{in}(t) = V_{in}e^{i\omega t}$ and $v_{out} = V_{out}e^{i\omega t}$, solve for the magnitude of V_{out} in terms of V_{in}, R, L, and C.

(b) In principle, this circuit could have a resonance if R is too big. Assuming that R is large enough that it can be ignored, estimate the resonant frequency, ω_0.

(c) If this circuit is intended to form a low-pass filter, what value of R would be needed to provide a gain of -3 db at the frequency calculated in part (b)?

(d) At high frequencies, how many deci-Bells per decade of frequency does this circuit attenuate? Compare this with the result for a first-order RC or RL low-pass filter.

3. Using the nodes listed on the circuit in the previous question and using the component values

$$\begin{aligned} L &= 10\ \mu\text{H} \\ C &= 2.53\ \text{nF} \\ R &= 44.4\ \Omega \end{aligned}$$

use SPICE to calculate the magnitude of the voltage gain as follows:

- The AC voltage source is described using
  ```
  Vxxx <N+> <N-> <DC offset> AC
  ```
  where you should set the DC offset to zero in this case.

- The magnitude of the voltage across the resistor is graphed using
  ```
  .PRINT AC VM(2)
  ```

- The frequency response is analysed using the `.AC` command:
  ```
  .AC DEC 10 1K 10MEG
  ```
  which will calculate the response at ten points per decade between the frequencies of 1 kHz and 10 MHz.

(a) Write the SPICE netlist that describes this circuit.
(b) Hand in a plot of the voltage gain across R as a function of frequency. At what frequency (in Hz) is the gain approximately equal to -3 db? How does this frequency compare with the calculation performed in question 2?
(c) Provide graphs of the voltage gain as a function of frequency when $R = 50\ \Omega$, $R = 100\Omega$ and $R = 1\ \text{k}\Omega$. Compare these frequencies with the results of the calculation performed in question 2.