Physics 536 - Assignment #1

1. Printed circuit boards are often manufactured adhering a very thin layer of copper to a sheet of epoxy impregnated fiberglass and etching away the unwanted copper to leave behind traces that form the conductors in a circuit. The thickness of the copper cladding is specified in *ounces per square foot.*

(a) Calculate the thickness in millimeters and 0.5-oz copper.

(b) Calculate the resistance at room temperature of a 0.5-oz copper trace that is 0.5 mm wide and 80 mm long.

(c) Flexible circuits can be fabricated using a process where silver ink is silk screened onto a flexible plastic sheet substrate. For an ink thickness of 15 μm, the resistivity is specified as $\rho' = 20$ mΩ/sq. Calculate the resistance of a silver print trace that is 0.5 mm wide and 80 mm long.

2. The following two-loop circuit has a constant current source, I, across which an unknown potential difference, ΔV, will be produced.
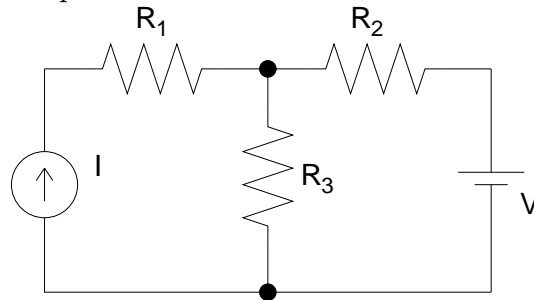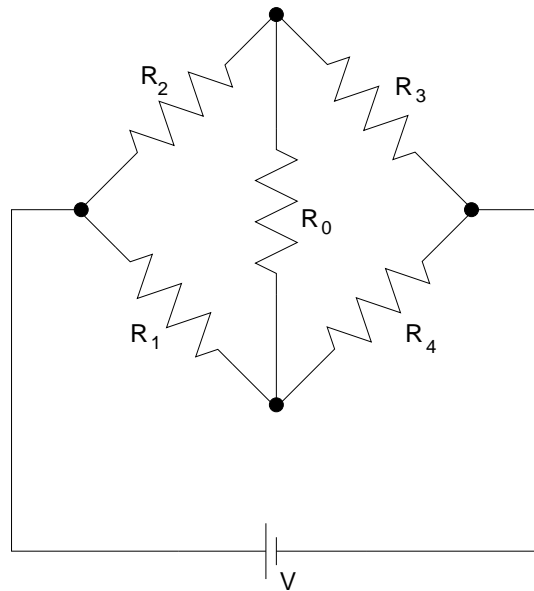

This circuit can still be analyzed as a linear 2×2 system of equations, which must be solved to determine the unknown ΔV and the current, I_2, that flows in the second loop with the voltage source.

(a) Write down the 2×2 system of linear equations in the unknowns ΔV and I_2.

(b) Use Kramer's rule to solve for the potential difference across the current source, ΔV.

(c) Use Kramer's rule to solve for the current, I_2, flowing in the second loop.

3. Not all resistor networks can be easily simplified using the formulas for resistances in series and parallel. The following circuit, which has five resistors in a "bridge" configuration, is one such circuit.

Nevertheless, it can still be analyzed as a three-loop circuit from which the current flowing through the voltage source can be calculated.

(a) Write down the 3×3 matrix equation which can be solved to determine the vector of currents in the three loops.

(b) Using Kramer's rule, express the current flowing through the voltage source as the ratio of determinants of two 3×3 matrices.

(c) By expanding the expression found in part (b), find an expression for the equivalent single resistance, $R_{\text{equiv.}}$ that would produce the same current flowing through the voltage source. To avoid a lot of tedious algebra, now would be a good time to learn how to use a symbolic manipulation program like Mathematica or Maple.

(d) Show that if $R_1 = R_2 \equiv R_{12}$ and $R_3 = R_4 \equiv R_{34}$, then the equivalent resistance does not depend on the value of R_0. Explain why this should be expected.

4. (a) Consider a non-ideal voltage source modeled as an ideal voltage source in series with an unknown resistance, R. Derive an expression for the resistance, R, in terms of the voltage, V, measured with no load and the voltage, V', measured across a load of resistance R_L.

(b) Consider a non-ideal voltage source modeled as an ideal current source in parallel with an unknown resistance, R. Derive an expression for the resistance, R, in terms of the voltage, V, measured with no load and the voltage, V', measured across a load of resistance R_L.

5. Consider the following circuit, which contains three loops:
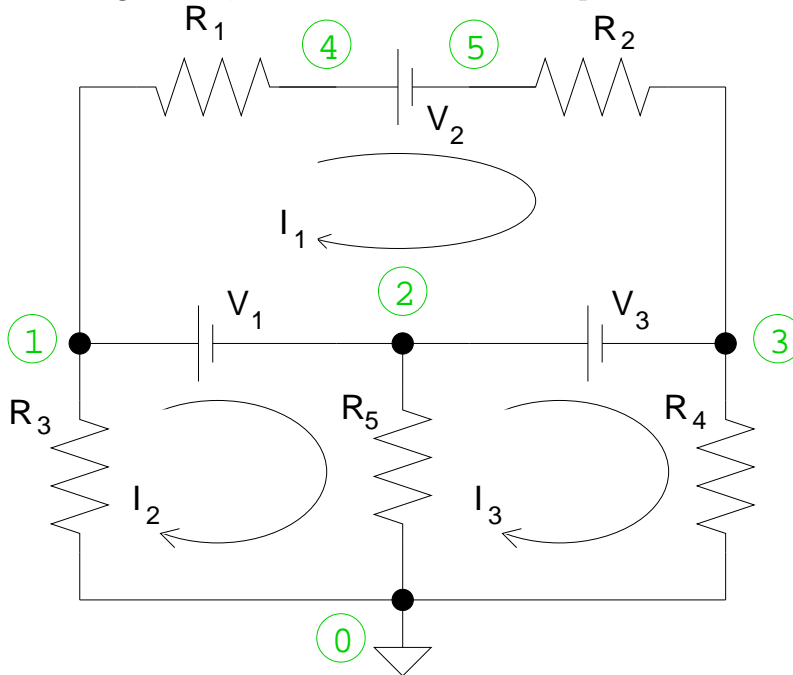
(a) Assign currents, I_1, I_2 and I_3 flowing clockwise through the three loops as shown. Write the 3×3 matrix equation that relates the currents, resistances and voltage sources.

(b) Solve for I_1, I_2 and I_3.

(c) Suppose the components in the circuit had the following values:

$$
\begin{aligned}
V_1 &= 5 \text{ V} \\
V_2 &= 10 \text{ V} \\
V_3 &= 5 \text{ V} \\
R_1 &= 10 \ \Omega \\
R_2 &= 20 \ \Omega \\
R_3 &= 5 \ \Omega \\
R_4 &= 10 \ \Omega \\
R_5 &= 10 \ \Omega
\end{aligned}
$$

Following the example provided at

http://www.physics.purdue.edu/~mjones/phys536/spice.html

write the SPICE netlist that describes this circuit using the node numbers indicated on the circuit diagram.