Physics 536 - Assignment #5 - Due February 22$^{\text{th}}$

1. Suppose a transmission line of length ℓ with characteristic impedance Z_0 was driven by a voltage source of the form $v_{in}(t) = Ve^{i\omega t}$ with a source impedance matching that of the transmission line.
(a) Show that the voltage measured at the source, $x = 0$, is

$$v(0,t) = \frac{Ve^{i\omega t}}{2}\left(1 - \Gamma_L e^{-2i\omega\ell/v}\right) \tag{1}$$

in which we assume that $\sqrt{(R' + i\omega L')(G' + i\omega C')} = i\omega/v$.
(b) Show that the magnitude of the voltage measured at the source is

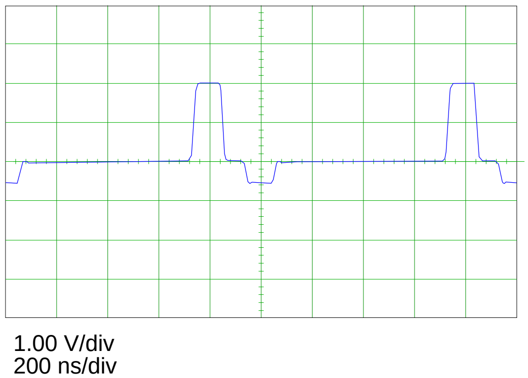
$$|V(0)| = \frac{V}{2}\sqrt{1 - 2\Gamma_L \cos 2\omega\ell/v + \Gamma_L^2} \tag{2}$$

(c) The *voltage standing wave ratio* (VSWR) is defined as the ratio $V_{\max}/V_{\min}$ where $V_{\max}$ and $V_{\min}$ are the maximum and minimum amplitudes of the voltage measured at the source as the frequency ω is varied over a wide range. Show that

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \tag{3}$$

and show how a measurement of the VSWR can be used to determine the absolute value of the reflection coefficient, $|\Gamma_L|$, and two possible values for the termination impedance, Z_L.

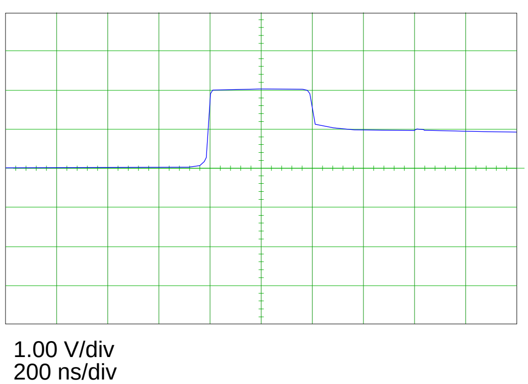
2. The following oscilloscope trace shows the voltage measured at the source ($x = 0$) end of a transmission line, where the source impedance matches the 50 Ω impedance of the transmission line. The input voltage is a pulse with an amplitude of 2 volts and a width of about 120 ns which is repeated every microsecond.



(a) How long does it take the pulse to reach the far end ($x = \ell$) of the transmission line? Assuming $v = 2/3c$, what is the physical length of the cable?

(b) What is the impedance of the load at $x = \ell$?

3. The following oscilloscope trace shows the voltage measured at the source ($x = 0$) end of a transmission line, where the source impedance matches the 50 Ω impedance of the transmission line. The input voltage is a step with an amplitude of 2 volts.



(a) What is the electrical length of the cable (in nanoseconds) and, assuming $v = 2/3c$, what is its physical length?

(b) What is the impedance of the load at $x = \ell$?