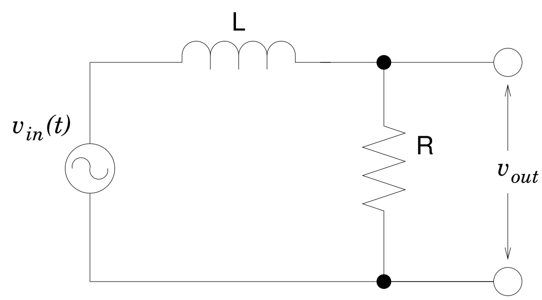
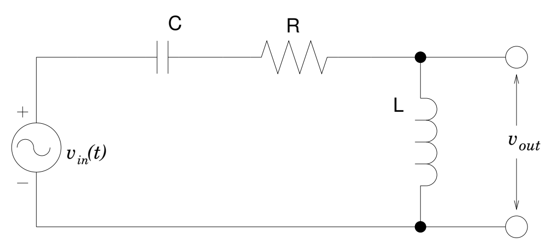
Physics 536 - Assignment #3 - Due February 8^{th}

1. Consider the low-pass RL filter circuit:



When $v_{in}(t) = V_{in}e^{i\omega t}$, the Thevenin equivalent circuit would consist of an ideal voltage source V_{Th} in series with an impedance Z_{Th}, both of which depend on the frequency, ω.
(a) With no load across V_{out}, calculate V_{out}. How does V_{out} behave in the low-frequency ($\omega \ll R/L$) and high-frequency ($\omega \gg R/L$) limits?
(b) Calculate the Thevenin equivalent impedance Z_{Th} for the circuit where the impedance of the resistor is R and the impedance of the inductor is $i\omega L$. How does Z_{Th} behave in the low-frequency and high-frequency limits?

2. A *second-order* high-pass filter is shown in the following circuit:

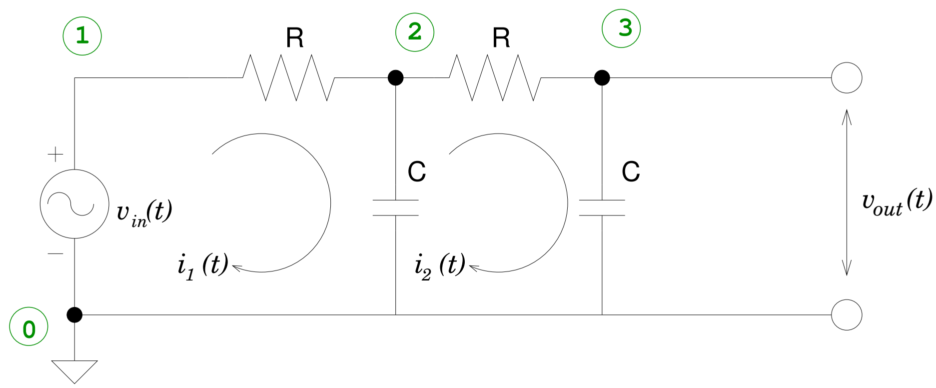


(a) When $v_{in}(t) = V_{in}e^{i\omega t}$ show that the voltage gain is

$$A = \frac{1}{\sqrt{\frac{R^2}{\omega^2 L^2} + (1 - \frac{1}{\omega^2 LC})^2}}$$

(b) In principle, this filter could act as an RLC resonant circuit. What is the resonant frequency? What value of R would give a voltage gain of $1/\sqrt{2}$ at this frequency?
(c) If $C = 0.1\ \mu$F, calculate the values of L and R that would be give a second-order high-pass filter with $f_{-3\text{ db}} = 100$ kHz. *Graph the gain in decibells as a function of frequency in Hz for these component values.*

3. A *second-order* low-pass filter is shown in the following circuit:



(a) Assuming $v_{in}(t) = V_{in}e^{i\omega t}$, $i_1(t) = I_1 e^{i\omega t}$, $i_2(t) = I_2 e^{i\omega t}$ and $v_{out}(t) = V_{out}e^{i\omega t}$, solve the system of equations to determine I_2 and show that

$$V_{out} = \frac{V_{in}}{1 + 3i\omega RC - \omega^2 R^2 C^2}$$

(b) Show that the voltage gain is given by the expression:

$$A = \frac{|V_{out}|}{|V_{in}|} = \frac{1}{\sqrt{1 + 7\omega^2 R^2 C^2 + \omega^4 R^4 C^4}}$$

(c) Show that when $\omega \gg 1/RC$, the gain in decibells decreases by 40 db per decade.
(d) Show that the frequency at which the gain falls to $G = -3$ db is given by

$$\omega^2_{-3\text{ db}} = \frac{1}{R^2C^2}\left(\frac{\sqrt{53} - 7}{2}\right)$$

The mangitude of the output voltage as a function of frequency can be calculated using SPICE by performing an *AC sweep analysis*. In this case, an independent AC voltage source is specified using

```
Vxxx <N+> <N-> <DC offset> AC
```

and the small-signal AC analysis is performed using an `.AC` command such as:

```
.AC DEC 6 1 10MEG
```

which will scan the frequency of independent AC voltage sources with 6 points per decade from 1 Hz to 10 MHz. The *magnitude* of the voltage at node 3, *ie,* $|V_{out}|$, is plotted using

```
.PRINT AC VM(3)
```

(e) Using the values $R = 60\ \Omega$ and $C = 10$ nF, use SPICE to produce a graph of the the voltage gain as a function of frequency over the range $1 < f < 10$ MHz. Indicate the position of the -3 db point and verify that it is at the frequency expected from part d.