

Physics 422 - Spring 2016 - Midterm Exam, March 10th

Answer all questions in the exam booklets provided.

*There are 6 questions - please answer **any five** of them.*

Explain your reasoning clearly but concisely.

(but it is not expected to be at the same level as the homework)

((although some of the homework is really pretty bad...))

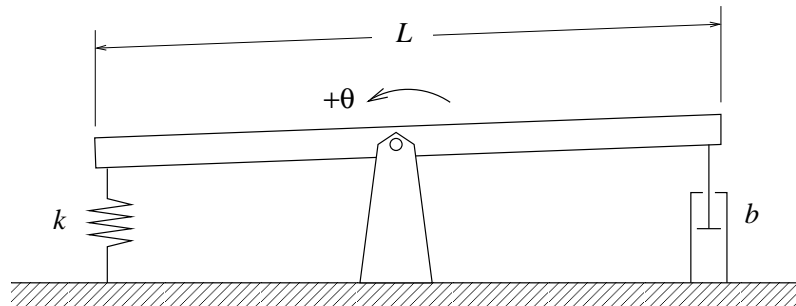
((just not yours...))

Clearly indicate which work is to be graded.

Each question is of equal weight.

You can use one page of your own notes/formulas.

1. A heavy beam, with length L and mass M , pivots about a fixed axis through its center and has a spring with spring constant k at one end and a mechanism that introduces a viscous damping force proportional to b at the other end as shown:



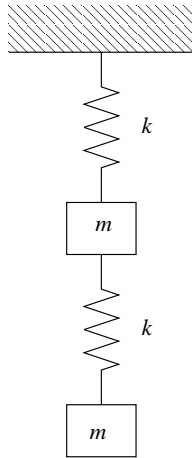
- (a) When in equilibrium, the beam is horizontal. Write the differential equation that describes the small angle θ measured with respect to the equilibrium position with $+\theta$ in the direction shown on the figure. The moment of inertia of the beam about the fixed axis is $I = \frac{1}{12}ML^2$.
- (b) Assuming the system is underdamped, what is the angular frequency of small oscillations about the equilibrium position?

2. A non-linear spring exerts a force that is a function of the distance stretched of the form

$$F(x) = -kx - ax^2$$

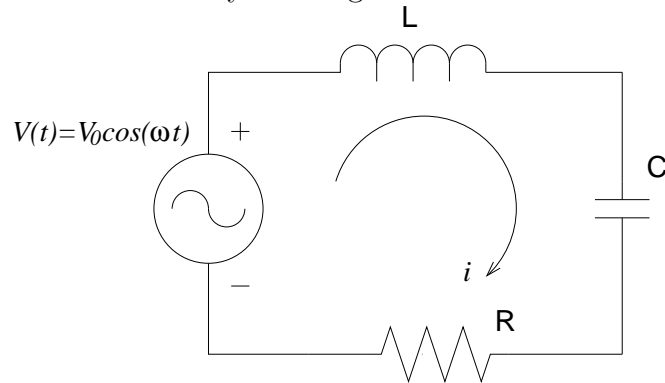
- (a) If a mass m is hung from the end of this spring, how far will it be stretched before it reaches a state of static equilibrium?
(b) What is the frequency of small oscillations about this equilibrium position?

3. Two identical masses are suspended from identical springs as shown



- (a) Write the equations of motion for the two masses in terms of their displacements from equilibrium, x_1 and x_2 , which can be defined in an inertial reference frame.
(b) Find the frequencies of the two normal modes of oscillation about the equilibrium positions.

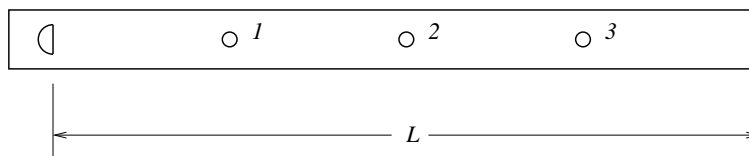
4. An RLC circuit is driven by a voltage source as shown below:



(a) Calculate the frequency, ω , of the voltage source that will maximize the power drawn by the circuit.

(b) Calculate the range of frequencies for which the power drawn by the circuit is at least half the maximal value.

5. A whistle consists of a tube of length L with three holes that are uniformly spaced along its length as shown.



The acoustic pressure will be zero at both ends of the tube and at any hole that is open to the atmosphere. If the speed of sound is v , what are the lowest frequency (not angular frequency) of the tone produced when

(a) All holes are closed

(b) Holes 1 and 3 are closed but hole 2 is open

(c) Holes 2 and 3 are closed but hole 1 is open

6. Consider a string with tension T on which N beads with equal mass m and spacing ℓ are attached. If all the beads are motionless at $t = 0$ then the motion of an arbitrary bead k can be described by a sum over normal modes of oscillation:

$$y_k(t) = \sum_{n=1}^N a_n \sin\left(\frac{nk\pi}{N+1}\right) \cos(\omega_n t)$$

where

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right).$$

Find an expression for a_n that will satisfy the initial value problem in which all masses are initially stationary and in their equilibrium position except for the first bead, described by $y_1(t)$, which is displaced by an amplitude A at $t = 0$.

It may be useful to recall that

$$\sum_{k=1}^N \sin\left(\frac{nk\pi}{N+1}\right) \sin\left(\frac{mk\pi}{N+1}\right) = \frac{N}{2} \delta_{nm}$$