# Assignment #8



1. When light undergoes total internal reflection, the reflection coefficients are

$$r_{\parallel} = -\frac{n_2^2 \cos\theta_i - i n_1 \sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_2^2 \cos\theta_i + i n_1 \sqrt{n_1^2 \sin^2\theta_i - n_2^2}}$$

$$r_{\perp} = \frac{n_1 \cos\theta_i - i n_1 \sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_1 \cos\theta_i + i n_1 \sqrt{n_1^2 \sin^2\theta_i - n_2^2}}$$

(a) The reflected intensity will be proportional to $|r|^2$ but since each reflection coefficient is a complex number of the form

$$r = \frac{x - iy}{x + iy}$$

$$|r|^2 = r^* r = \left(\frac{x - iy}{x + iy}\right)^* \left(\frac{x - iy}{x + iy}\right)$$

$$= \left(\frac{x + iy}{x - iy}\right)\left(\frac{x - iy}{x + iy}\right)$$

$$= \frac{x^2 + y^2}{x^2 + y^2} = 1 .$$

(b) When $n_1 = 1.5$ and $n_2 = 1$ and $\theta_i = 53.3°$, we will have

$$n_2^2 \cos\theta_i = 0.5976$$
$$n_1 \cos\theta_i = 0.8964$$
$$n_1\sqrt{n_1^2 \sin^2\theta_i - n_2^2} = 1.0022$$
$$\sqrt{n_1^2 \sin^2\theta_i - n_2^2} = 0.6681$$

Then, if we write $r = e^{i\varphi} = \dfrac{x - iy}{x + iy}$

we will have $\tan\varphi = \dfrac{-2xy}{x^2 - y^2}$

So $\tan\varphi_{\parallel} = \dfrac{-2(0.5976)(1.0022)}{(0.5976)^2 - (1.0022)^2} = 1.8506$

$$\Rightarrow \varphi_{\parallel} = 61.61°$$

$\tan\varphi_{\perp} = \dfrac{-2(0.8964)(0.6681)}{(0.8964)^2 - (0.6681)^2} = -3.3535$

$$\Rightarrow \varphi_{\perp} = -73.40°$$

The phase difference is

$$\Delta\varphi = 61.61° + 73.40° = 135°$$

There are two internal reflections, so the total phase difference is

$$2\Delta\varphi = 2 \times 135° = 270° = 3\pi/2$$

(c) This is $3/4\,\lambda$ so the emerging light will be circularly polarized.

But! Is it left- or right circular polarized?

over ...

(c) cont.

If the rhomb is oriented so that the plane of incidence is in the $x$-$z$ plane, then the $\hat{\imath}$ direction corresponds to the $\perp$ and the $\hat{\jmath}$ direction corresponds to the $\parallel$ components.

If the incident light is linearly polarized at $45^0$ then it can be represented by

$$\vec{E}(z,t) = E_0\left(\hat{\imath}\cos(kz-\omega t) + \hat{\jmath}\cos(kz-\omega t)\right)$$

When it emerges, it is of the form

$$\vec{E}'(z,t) = E_0\left(\hat{\imath}\cos(kz-\omega t+2\varphi_\perp) + \hat{\jmath}\cos(kz-\omega t+2\varphi_\parallel)\right)$$

which is equivalent to

$$\vec{E}'(z,t) = E_0\left(\hat{\imath}\cos(kz-\omega t) + \hat{\jmath}\cos(kz-\omega t+2(\varphi_\parallel-\varphi_\perp))\right)$$

But $\varphi_\parallel - \varphi_\perp = 135^0$ so this is

$$\vec{E}'(z,t) = E_0\left(\hat{\imath}\cos(kz-\omega t) + \hat{\jmath}\cos(kz-\omega t+3\pi/2)\right)$$

$$= E_0\left(\hat{\imath}\cos(kz-\omega t) + \hat{\jmath}\sin(kz-\omega t)\right)$$

So the emerging light is right-circular polarized.

If the incident light was of the form

$$\vec{E}(z,t) = E_0\left(\hat{\imath}\cos(kz-\omega t) - \hat{\jmath}\cos(kz-\omega t)\right)$$

then the emerging light would be left-circular polarized.

2. Two incoherent light beams are described by Stokes parameters

$$\vec{S}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{S}_2 = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 3 \end{pmatrix}$$

(a) The first beam is linearly polarized along the horizontal axis.

The second beam is right-circular polarized and has three times the intensity of the first beam.

(b) The resulting Stokes parameters are

$$\vec{S}' = \vec{S}_1 + \vec{S}_2 = \begin{pmatrix} 4 \\ 1 \\ 0 \\ 3 \end{pmatrix}$$

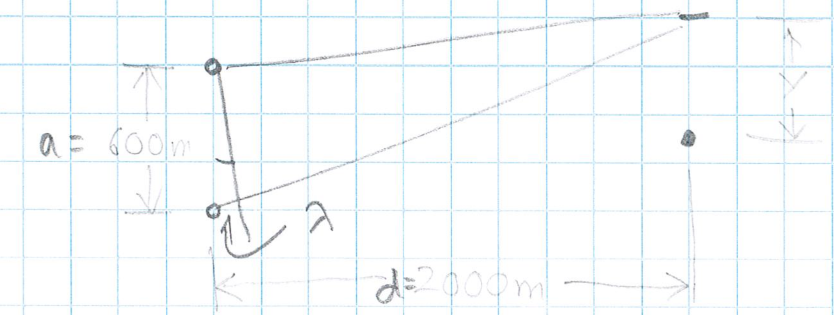
which corresponds to elliptically polarized light with a horizontal major axis.

(c) The degree of polarization is

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \frac{\sqrt{1+9}}{4} = \frac{\sqrt{10}}{4} = 0.79.$$

(d) When incoherent beams of light with stokes parameters $(1, 1, 0, 0)$ and $(1, -1, 0, 0)$ the result is incoherent unpolarized light with twice the intensity of the individual beams.

3.




The question is asking for the distance, y, at which the first maximum occurs.

The wavelength is $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{10^6 \text{ s}^{-1}}$

$= 299.8 \text{ m}$

Then if $d = 2000$ m, the distance $y$ will satisfy

$$\frac{y}{d} = \frac{\lambda}{a} \Rightarrow y = \frac{\lambda d}{a} = \frac{(299.8\text{ m})(2000\text{m})}{(600\text{ m})}$$

$$\approx 1000 \text{ m}.$$

4. When the mirror is moved a distance $d$, the optical path length changes by $2d$.

Thus, $\lambda = \dfrac{2 \cdot (0.0225 \text{ mm})}{100}$

$= 0.00045$ mm

$= 450$ nm (indigo).