YOUR IMAGINATION™

Blue Book

EXAMINATION BOOK

BOX NO. ______

NAME ______
SUBJECT PHYS 422
CLASS ______
SECTION ______
INSTRUCTOR ______
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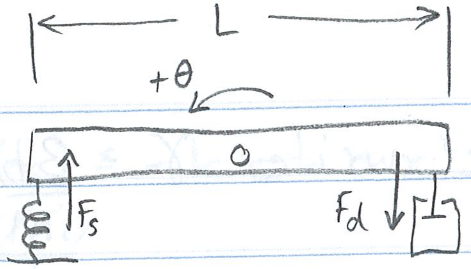


$$F_s = k\frac{L\theta}{2} \qquad F_d = b\frac{L\dot{\theta}}{2}$$

(directions as indicated on the free-body diagram)

Torques:

$$N_s = -\frac{kL^2\theta}{4} \qquad N_d = -\frac{bL^2\dot{\theta}}{4}$$

Equation of motion:

$$I\ddot{\theta} = \sum N = -\frac{kL^2}{4}\theta - \frac{bL^2}{4}\dot{\theta}$$

or

$$\frac{ML^2}{12}\ddot{\theta} + \frac{bL^2}{4}\dot{\theta} + \frac{kL^2}{4}\theta = 0$$

or $$\ddot{\theta} + \frac{3b}{M}\dot{\theta} + \frac{3k}{M}\theta = 0$$

If we write $\gamma = \frac{3b}{M}$

and $\omega_0^2 = \frac{3k}{M}$

then the equation of motion is

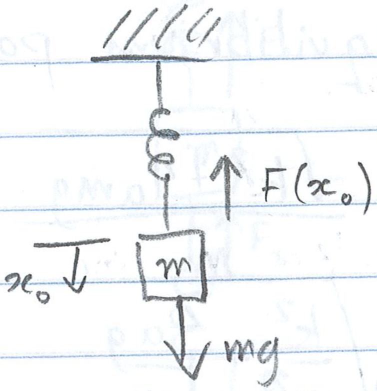
$$\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2\theta = 0$$

The frequency of free oscillations is

$$\omega = \sqrt{\omega_0^2 - \gamma^2/4}$$

$$= \sqrt{\frac{3k}{M} - \frac{9b^2}{4M^2}}$$

$$F(x) = -kx - ax^2$$



$$kx_0 + ax_0^2 = mg$$

$$ax_0^2 + kx_0 - mg = 0$$

$$x_0 = \frac{-k \pm \sqrt{k^2 + 4amg}}{2a}$$

$$= \frac{-k}{2a} + \sqrt{\frac{k^2}{4a^2} + \frac{mg}{a}}$$

we choose the positive root so that $x_0 > 0$.

$$F(x_0 + \delta x) = F(x_0) + \left.\frac{dF}{dx}\right|_{x_0} \delta x$$

$$\left.\frac{dF}{dx}\right|_{x_0} = -k - 2ax_0$$

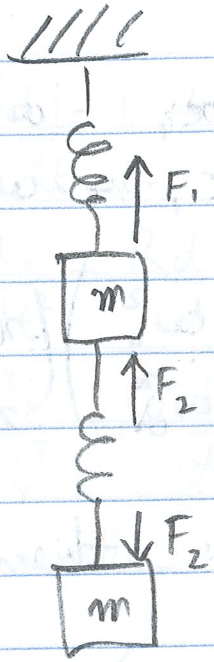
$$= -k + k - \sqrt{k^2 + 4amg}$$

$$= -\sqrt{k^2 + 4amg}$$

Frequency of small oscillations about the equilibrium position x_0 is

$$\omega_0^2 = \frac{k'}{m} = \frac{\sqrt{k^2+4amg}}{m}$$

$$= \sqrt{\frac{k^2}{m^2}+\frac{4ag}{m}}$$



$$F_1 = -kx_1$$
$$F_2 = -k(x_1 - x_2)$$

$$m\ddot{x}_1 + kx_1 + k(x_1 - x_2) = 0$$
$$\ddot{x}_1 + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0$$

$$m\ddot{x}_2 + k(x_2 - x_1) = 0$$
$$\ddot{x}_2 + \omega_0^2 x_2 - \omega_0^2 x_1 = 0$$

$$\ddot{x}_1 + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0$$
$$\ddot{x}_2 + \omega_0^2 x_2 - \omega_0^2 x_1 = 0$$

Assuming $x_1 = A\cos\omega t$
$x_2 = B\cos\omega t$

$$\ddot{x}_1 = -\omega^2 x_1$$
$$\ddot{x}_2 = -\omega^2 x_2$$

$$-\omega^2 x_1 + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0$$
$$-\omega^2 x_2 - \omega_0^2 x_1 + \omega_0^2 x_2 = 0$$

$$\begin{pmatrix} -\omega^2 + 2\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & -\omega^2 + \omega_0^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

Let $\lambda = \omega^2$. then

$$(\omega_0^2 - \lambda)(2\omega_0^2 - \lambda) - \omega_0^4 = 0$$

$$\lambda^2 - 3\omega_0^2 \lambda + \omega_0^4 = 0$$

$$\lambda = \frac{3\omega_0^2}{2} \pm \frac{1}{2}\sqrt{9\omega_0^4 - 4\omega_0^4}$$

$$\omega^2 = \omega_0^2\left(\frac{3}{2} \pm \frac{\sqrt{5}}{2}\right)$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = V(\omega)$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = -\omega V_0 \sin \omega t$$

$$\frac{d^2i}{dt^2} + \gamma\frac{di}{dt} + \omega_0^2 i = -\omega_0 V_0 \sin \omega t$$

where $\gamma = \frac{R}{L}$ $\quad \omega_0^2 = \frac{1}{LC}$.

Maximum power when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$$FWHM = \gamma = \frac{R}{L}$$

(a) With all holes closed the boundary conditions allow waves with $\lambda = 2L$.

But $f = v/\lambda = \dfrac{v}{2L}$.

b) If there is a pressure node at $L/2$ then $\lambda = L$ and $f = \dfrac{v}{L}$.

c) If there is a pressure node at hole 1 then the lowest frequency would correspond to $\dfrac{\lambda}{2} = \dfrac{L}{4} \Rightarrow \lambda = \dfrac{L}{2}$

$$f = \frac{2v}{L}$$

$$y_k(t) = \sum_{n=1}^{N} a_n \sin\left(\frac{nk\pi}{N+1}\right) \cos(\omega_n t)$$

$$y_k(0) = \sum_{n=1}^{N} a_n \sin\left(\frac{nk\pi}{N+1}\right) = A\,\delta_{1k}$$

But,

$$\sum_{n,k} a_n \sin\left(\frac{nk\pi}{N+1}\right) \sin\left(\frac{mk\pi}{N+1}\right) = A \sum_{k} \delta_{1k} \sin\left(\frac{mk\pi}{N+1}\right)$$

$$\sum_{n} a_n \delta_{nm} \cdot \frac{N}{2} = A \sin\left(\frac{m\pi}{N+1}\right)$$

$$a_m \cdot \frac{N}{2} = A \sin\left(\frac{m\pi}{N+1}\right)$$

$$a_m = \frac{2A}{N} \sin\left(\frac{m\pi}{N+1}\right)$$