Physics 42200 Assignment # 1 1
Spring 2016

1. (a) Find the roots of $(x-2)(x+3)=0$.

That's easy! They are obviously

$$x=2$$
and
$$x=-3.$$

(b) Find the roots of $x^3-7x+6=0$.

That's a bit harder. It would be legitimate to use Mathematica but you might also try to guess one root. Try $x=2$ ...

$$2^3-7\cdot 2+6=8-14+6=0$$

So $x=2$ is a root and we know that $(x-2)$ is a factor of the polynomial. We can reduce it to a quadratic equation by synthetic division:

$$\begin{array}{r|l} & x^2+2x-3 \\ \hline x-2 & x^3-7x+6 \\ & x^3-2x^2 \\ \hline & 2x^2-7x+6 \\ & 2x^2-4x \\ \hline & -3x+6 \\ & -3x^2+6 \\ \hline & 0 \end{array}$$

But we recognize how to factor $x^2+2x-3=0$. It is just $(x+3)(x-1)=0$ so the roots are

$$x=2$$
$$x=-3$$
$$x=1.$$

There is also a formula for the roots of a third-order polynomial that you could look up.

(c) Find the roots of $x^2 - 2x + 2 = 0$. 2

You can also just use the quadratic formula: When $Ax^2 + Bx + C = 0$ the roots are

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

In this case, $A = 1$, $B = -2$, $C = 2$

So the roots are
$$x = \frac{2 \pm \sqrt{4-8}}{2}$$
$$= 1 \pm \frac{\sqrt{-4}}{2}$$
$$= 1 \pm \sqrt{-1}$$
$$= 1 \pm i.$$

The roots are complex numbers in this case.

2. This exercise is just to check that you remember how to calculate determinants.

(a)
$$\begin{vmatrix} 1-x & 1 \\ 4 & -2-x \end{vmatrix} = 0$$

$$(1-x)(-2-x) - 4 = (x-1)(x+2) - 4$$
$$= x^2 + x - 6 = 0$$

But $x^2 + x - 6 = (x+3)(x-2) = 0$

So $x = 2$ or $x = -3$.

(b)
$$\begin{vmatrix} 0 & 1-x & 0 \\ 1-x & 0 & 1 \\ 4 & 0 & -2-x \end{vmatrix} = 0$$

$$-(1-x)[(1-x)(-2-x) - 4] = 0$$
$$(x-1)(x-1)(x+2) - 4(x-1) = 0$$
$$(x^2-2x+1)(x+2) - 4x + 4 = 0$$
$$x^3 - \cancel{2x^2} + x + \cancel{2x^2} - 4x + 2 - 4x + 4 = 0$$
$$x^3 - 7x + 6 = 0$$

But we already worked out that
$$x^3 - 7x + 6 = (x-2)(x+3)(x-1) = 0$$

So the solutions are $x = 2$
$x = -3$
$x = 1$.

(c) $$\begin{vmatrix} 1+2i-x & 1 \\ 3 & 1-2i-x \end{vmatrix} = 0$$

$$(1+2i-x)(1-2i-x) - 3 = 0$$

$$(1+2i)(1-2i) - 2x + x^2 - 3 = 0$$

$$5 - 2x + x^2 - 3 = 0$$

$$x^2 - 2x + 2 = 0$$

and again, we already found the roots to be

$$x = 1+i$$
and $$x = 1-i$$

3. $$x(t) = A\cos\omega t + B\sin\omega t$$

(a) $$x(t) = \mathrm{Re}[r e^{i(\omega t - \varphi)}]$$
$$= r\cos(\omega t - \varphi).$$

These must be equal at all times $t$. In particular, at $t=0$,

$$x(t=0) = A = r\cos(-\varphi) = r\cos\varphi.$$

Furthermore, when $\omega t = \pi/2$,

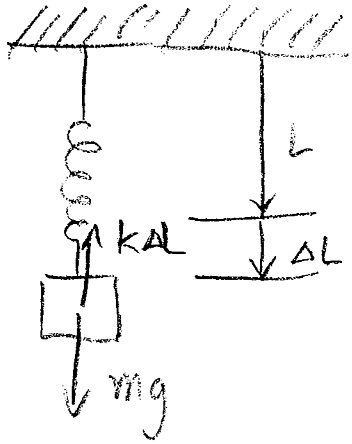
$$x(t = \tfrac{\pi}{2\omega}) = B = r\cos\left(\tfrac{\pi}{2} - \varphi\right)$$
$$= -r\sin(-\varphi) = r\sin\varphi.$$

(b) $$r^2\cos^2\varphi + r^2\sin^2\varphi = r^2 = A^2 + B^2$$

Thus, $r = \pm\sqrt{A^2 + B^2}$

Also, $$B/A = \frac{r\sin\varphi}{r\cos\varphi} = \tan\varphi$$
$$\Rightarrow \varphi = \tan^{-1}(B/A)$$

4.




When in equilibrium, $mg - k\Delta L = 0$

Therefore, $k = \dfrac{mg}{\Delta L}$

(a) The frequency of free oscillations is

$$\omega = \sqrt{k/m} = \sqrt{g/\Delta L}$$

(b) If the same system was placed on the surface of the moon the frequency would remain unchanged because $k$ is a property of the spring, not the environment.

So, even though $g' = \dfrac{g}{6}$, the frequency is still

$$\omega = \sqrt{\frac{g}{\Delta L}}.$$