Physics 422 - Spring 2016 - Assignment #6, Due April 1st

1. A transmission line has length L with characteristic impedance Z_0 in which signals propagate with speed v. One end is attached to a voltage source with source impedance $Z_s = Z_0$ while the other end is attached to a load with unknown impedance Z_l.

(a) What is the reflection coefficient at the end of the transmission line attached to the load?

(b) What is the reflection coefficient at the end of the transmission line attached to the source?

(c) If the voltage source produces an incident wave that has the form

$$V(t) = V_0 \cos(\omega t)$$

at the end of the transmission line attached to the source, write an expression for the incident wave at any point z along the transmission line.

(d) Write an expression for the reflected wave at any point z in the transmission line.

(e) The voltage measured at the source end of the transmssion line would be the sum of incident and reflected waves. Write an expression for the voltage measured at the source end as a function of frequency ω.

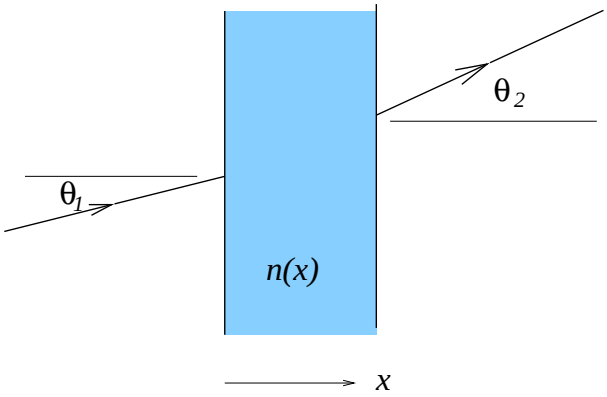
(f) The Voltage Standing Wave Ratio (VSWR) is the ratio of the maximum voltage to the minimum voltage measured at the source as the frequency ω is varied. Show how a measurement of the VSWR can be used to determine the load impedance Z_l.

2. Consider a rectangular prism with dimensions $L_x \times L_y \times L_z$ containing a gas in which sound propagates with velocity v. If the displacement from equilibrium of the gas molecules vanishes on the boundaries of the prism, show that the allowed frequencies are

$$\omega_{n_1 n_2 n_3} = v\sqrt{\left(\frac{n_1\pi}{L_x}\right)^2 + \left(\frac{n_1\pi}{L_y}\right)^2 + \left(\frac{n_1\pi}{L_z}\right)^2}$$

where n_1, n_2, and n_3 are integers.

3. Suppose a ray of light is incident on a transparent sheet of material of thickness w with an index of refraction that varies as a function of x as shown.



If the index of refraction of the medium on the left and the right of the sheet is the same, show that $\theta_2 = \theta_1$, independent of the function $n(x)$.