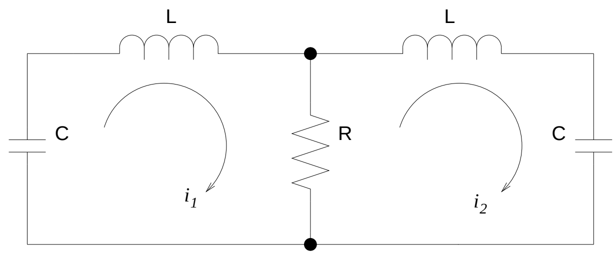
Physics 422 - Spring 2016 - Assignment #5, Due February 26$^{\text{th}}$

1. Consider the two-loop circuit shown below:



(a) Write the coupled set of differential equations for the currents, $i_1(t)$ and $i_2(t)$, which flow in each loop.

(b) Assuming the currents are of the form $i_1(t) = Ae^{\alpha t}$ and $i_2(t) = Be^{\alpha t}$, write the algebraic system of equations satisfied by $i_1(t)$ and $i_2(t)$.

(c) Write the system of equations as a matrix equation and find the characteristic polynomial that α must satisfy in order to satisfy the equality.

(d) In the case when $i_1(t) = i_2(t)$, explain why you would expect $\alpha = \pm i/\sqrt{LC}$ to be a root of the characteristic polynomial.

(e) Use synthetic division, or whatever method you prefer, to find the other roots of the polynomial.

(f) What are the frequencies of the two normal modes of oscillation?

2. Consider a string of length L with tension T and mass per unit length μ. The string is fixed at $x = 0$ and $x = L$, the initial displacement of the string at time $t = 0$ is $f(x)$ and the initial velocity is $g(x)$. The general solution to the initial value problem can be written

$$y(x,t) = \sum_{n=1}^{\infty} \left(a_n \sin\left(\frac{n\pi x}{L}\right) \cos(\omega_n t) + b_n \sin\left(\frac{n\pi x}{L}\right) \sin(\omega_n t) \right)$$

where $\omega_n = n\pi v/L$ with $v = \sqrt{T/\mu}$. Write expressions for a_n and b_n in terms of the initial displacement $f(x)$ and the initial velocity $g(x)$.

3. A pulse on a string is shaped like a parabola, and at $t = 0$ is described by the function

$$f(x) = \begin{cases} 0 & x < 2/5 \\ 1 - 100\left(x - \frac{1}{2}\right)^2 & 2/5 < x < 3/5 \\ 0 & x > 3/5 \end{cases}$$

If the pulse moves in the $+x$ direction with velocity v, what is the initial transverse velocity of the string, as a function of x?