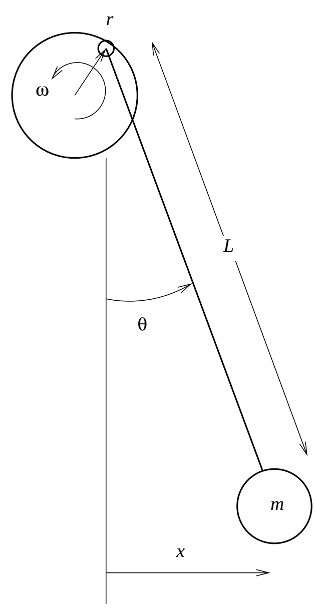
Physics 422 - Spring 2016 - Assignment #4, Due February 12$^{\text{th}}$

1. A mass m forms a pendulum of length L, and is hung from a pin attached to a wheel of radius r that rotates with constant angular velocity ω as shown:



(a) Consider the axis of the wheel to be an inertial reference frame. Write expressions for the coordinates of the pin in this reference frame, $X(t)$ and $Y(t)$, as functions of time.

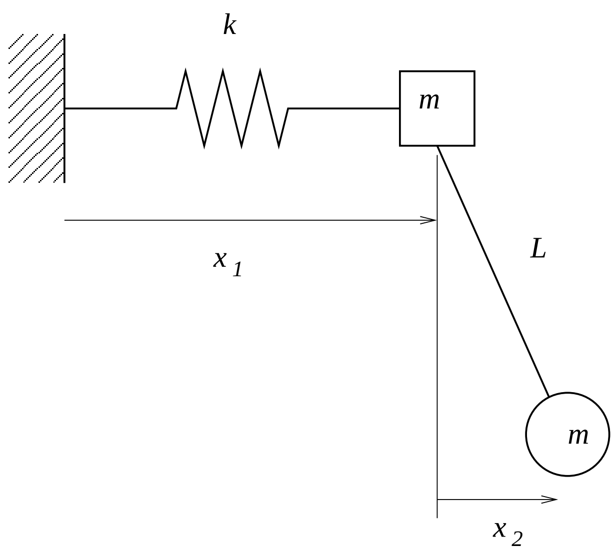
(b) It is convenient to describe the position of the mass m in terms of the angle $\theta(t) \approx x(t)/L$ which is defined in the non-inertial reference frame for which the pin is the origin. Write an expression for the horizontal position of the mass *in the inertial reference frame* in terms of $X(t)$ and $x(t)$. Also, explain why the motion $y(t)$ is not very interesting, when θ is a small angle.

(c) The only real forces that act on the mass is gravity and the tension in the string that attaches the mass to the pin on the wheel. Describe the horizontal acceleration of the mass in terms of the real forces and the coordinates $X(t)$ and $x(t)$.

(d) It is observed that when the wheel is not turning, the amplitude of horizontal oscillations decreases by $1/e$ in time τ. Write the differential equation for $x(t)$ for the case when the wheel is not turning, and for the case where the wheel is turning with angular velocity ω. Describe the amplitude and phase (relative to the position of the pin) of the steady state motion $x(t)$.

(e) What is the Q-value for this oscillator? Show that the amplitude of horizontal oscillations when $\omega = \omega_0 = \sqrt{g/L}$ is rQ.

2. A mass m is attached to a spring, and a second equal mass is hung from the first mass by a string of length L, forming a pendulum as shown:



(a) Write the set of coupled differential equations for $x_1(t)$ and $x_2(t)$. Notice that $x_2(t)$ is described in a non-inertial reference frame.

(b) If the spring constant, $k = gm/L$, show that the frequencies of the normal modes of oscillation are

$$\omega^2 = \frac{g}{2L}\left(3 \pm \sqrt{5}\right)$$