**Physics 422 - Spring 2016 - Assignment #3, Due February 5th**

**1.** In Assignment #2, you found that a conical float of length $L$ attached to a mass $m$ experiences a net force

$$F(u) = \rho g V_0(1 - 3u + 3u^2 - u^3) - mg$$

where $u = y/L$ and $V_0$ is the total volume of the float.
(It is convenient to describe the motion in terms of the dimensionless quantity $u$, because the description is then independent of the size of the physical system. The properties of any real system can then be obtained by multiplying $u$ by $L$.)

**(a)** Sketch the phase diagram (*ie.* points in the $\dot{u}$ *vs* $u$ plane) for small oscillations about the equilibrium position, $u_0$, for three different total energies, $E$.

**(b)** Sketch the phase diagram for larger oscillations for three energies ephasizing the deviations from the motion for small oscillations. Remember, because it's a cone, the force is greater than a simple linear force as the float sinks below the equilibrium position, but less than a simple linear force as it rises above the equilibrium position.

**(c)** If the float has sufficient energy that it can leap entirely out of the water, or be entirely submerged below the water then in either of these two cases, the force acting on it will be constant. Sketch the phase diagram for this case.

**2.** A block of mass $m$ slides on a horizontal frictionless surface and is connected to a spring which has a spring constant $k$. The other end of the spring is fixed. There is also a viscous damping mechanism attached to the mass. It is observed that when the spring is compressed by a distance $d$, the spring force is equal to the damping force observed when the mass moves with a speed $u$.

**(a)** For this complete system, including both the spring and the damper, write the differential equation governing horizontal oscillations of the mass in terms of $k$, $d$, $m$, and $u$.

**(b)** For special case where $u/d = \sqrt{k/m}$, calculate the ratio of the oscillation frequency to the frequency of undamped oscillations, $\omega_0 = \sqrt{k/m}$.

**(c)** Calculate the $Q$-value for this system.