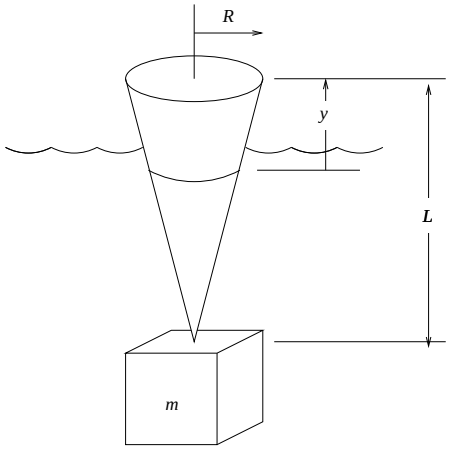
Physics 422 - Spring 2016 - Assignment #2, Due January 29th

1. A conical float has a total length, L, and its base has a radius, R and has a mass m hanging from the submerged end as shown. If the float is filled with air, its weight will be negligible compared with the weight of water it displaces.



(a) Calculate the volume of water displaced when the top of the float is located a distance y above the surface of the water.

(b) Calculate the distance y_0 when the float is in static equilibrium. Here are some suggestions...

1. Show that the volume of water displaced can be written

$$V(y) \;=\; V_0(1 - 3u + 3u^2 - u^3)$$

   where V_0 is the total volume of the float and $u = y/L$.

2. Show that when the system is in equilibrium,

$$1 - 3u + 3u^2 - u^3 \;=\; m/M$$

   where M is the mass of water that would be displaced by the entire volume of the float.

3. Look up a formula for the roots of a third-order polynomial and solve for u and then y_0.

(c) If the float is displaced by a small distance Δy from the equilibrium position, so that $y = y_0 + \Delta y$, calculate the change in the buoyant force, *to first order* in Δy.

(d) Show that the frequency of small oscillations about the equilibrium position is

$$\omega^2 = \frac{3g}{L}\left(\frac{M}{m}\right)^{1/3}$$

For this problem, it is okay to check your work using symbolic manipulation package like Mathematica, but you must be sure to show each step in your algebra.