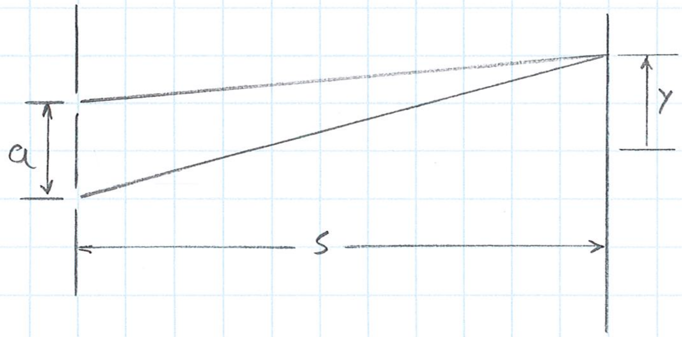
Assignment # 8 Solutions .

1.



Bright fringes occur when $a\sin\theta = m\lambda$
or when

$$y = \frac{m\lambda s}{a}$$

When red light with $\lambda_0 = 780$ nm in the first order fringe overlaps with violet in the second order fringe we have

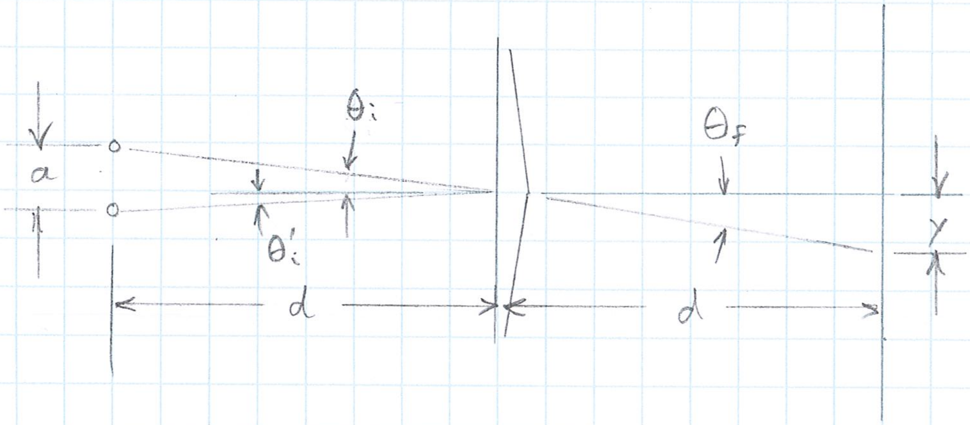
$$y = \frac{\lambda_0 s}{a} = \frac{2\lambda s}{a}$$

Therefore, $\lambda = \frac{\lambda_0}{2} = 390$ nm .

2. The Fresnel biprism is equivalent to a double slit experiment with slit spacing given by

$$a/2 = d(n-1)\,\alpha$$

In this case, the geometry looks like this.



where $\theta_f = \theta_i - (n-1)\alpha = \theta_i' + (n-1)\alpha$

Bright fringes occur when $a \sin\theta_i = m\lambda$
or $a\theta_i = m\lambda$
when $\theta_i \ll 1$.

Hence, $\frac{1}{2}a(\theta_i + \theta_i') = a\,\theta_f = m\lambda$

and bright fringes will be located at $y = d\,\theta_f = \dfrac{d\,m\lambda}{a}$

But $a = 2d(n-1)\alpha$ so $y = \dfrac{m\lambda}{2(n-1)\alpha}$

and $\alpha = \dfrac{\lambda}{2(n-1)\Delta y} = \dfrac{500\text{ nm}}{2(1.5-1)(0.5\text{mm})} = 0.001$

$= 0.06^\circ$

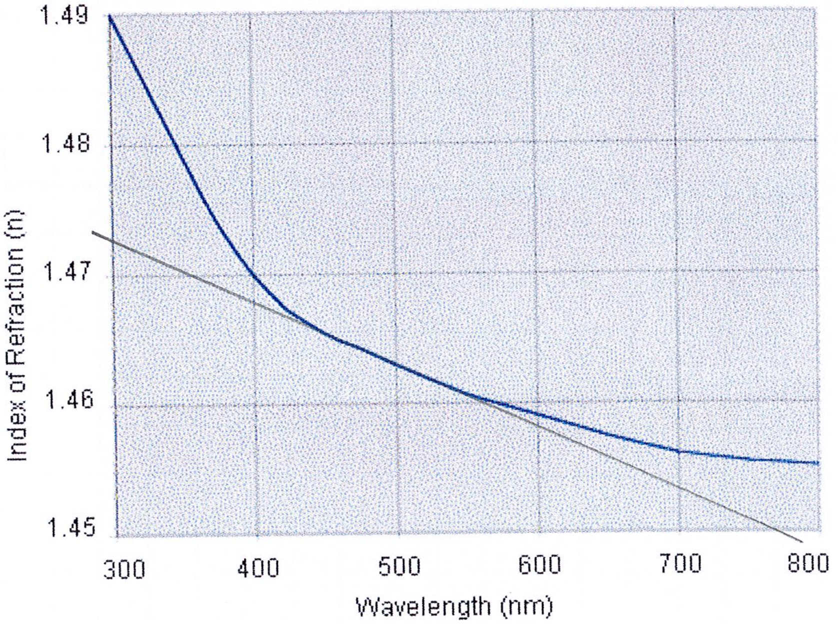
3. Resolving power for a prism is

$$R = \frac{\lambda}{\Delta\lambda} = \frac{B\,dn}{d\lambda}$$

where B is the base length.

(a) For quartz,

http://www.instant-analysis.com/Principles/refraction.htm



For wavelengths between approximately 430 nm and 550 nm, the first derivative is

$$\frac{dn}{d\lambda} = \frac{1.473 - 1.450}{300\,\text{nm} - 770\,\text{nm}} = -0.000049\ \text{nm}^{-1}.$$

When $B = 10$ cm, $R = (10\times10^{-2}\,\text{m})(0.000049\times10^{9}\,\text{m}^{-1})$
$= 4900$

(b) For a Fabrey-Perot interferometer,

$$R = \frac{\lambda}{\Delta\lambda} = \tilde{\mathcal{F}} \cdot \frac{2n_f d}{\lambda_0}$$

Typical values are $\tilde{\mathcal{F}} = 50$, $n_f d = 1$ cm.

Then, $$R = \frac{50 \cdot 2 \cdot 10^{-1}\,\text{m}}{500 \times 10^{-9}\,\text{m}} = 2 \times 10^{6}$$

(c) The sodium doublet consists of two spectral lines with a mean wavelength of

$$\lambda = \tfrac{1}{2}(588.9950\ \text{nm} + 589.5924\ \text{nm})$$
$$= 589.2937\ \text{nm}$$

and a spacing of $\Delta\lambda = 589.5924 - 588.9950$
$= .5974$ nm.

The minimum resolvable wavelength difference with the prism is

$$(\Delta\lambda)_{min} = \frac{\lambda}{R} = \frac{589.3\,\text{nm}}{4900} = 0.12\ \text{nm}.$$

The minimum resolvable wavelength difference with the Fabrey-Perot interferrometer is

$$(\Delta\lambda_{min}) = \frac{\lambda}{R} = \frac{589.3\,\text{nm}}{2 \times 10^{6}} = 0.00029\ \text{nm}.$$

The doublet would be resolvable with the Fabrey-Perot interferometer but would be barely resolvable with the prism.