Assignment #6

1. The focal length of the first thin lens is

$$\frac{1}{f_1} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $n_e = 1.5$, $R_1 = 10\,cm$ and $R_2 = -10\,cm$.

$$\Rightarrow \quad \frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{10cm} + \frac{1}{10cm}\right) = \frac{1}{10\,cm}$$

So $f_1 = 10\,cm$.

The focal length of the second lens is

$$\frac{1}{f_2} = (1.5 - 1)\left(\frac{-1}{20cm} - \frac{1}{20cm}\right) = \frac{-1}{20cm}$$

So $f_2 = -20\,cm$.

The image formed by the first lens is located at

$$\frac{1}{s_{i_1}} = \frac{1}{f_1} - \frac{1}{s_o} = \frac{1}{10\,cm} - \frac{1}{50\,cm} = \frac{2}{25\,cm}.$$

So $s_{i_1} = 12.5\,cm$.

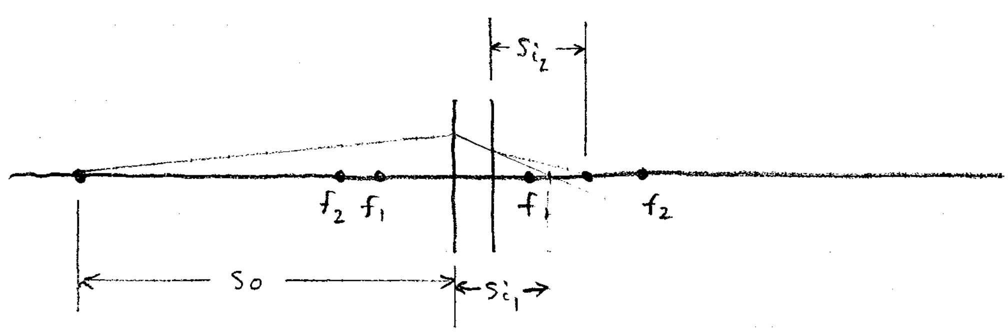
This image forms a virtual object for the second lens which is located at

$$s_{o_2} = d - s_{i_1} = 5\,cm - 12.5\,cm = -7.5\,cm$$

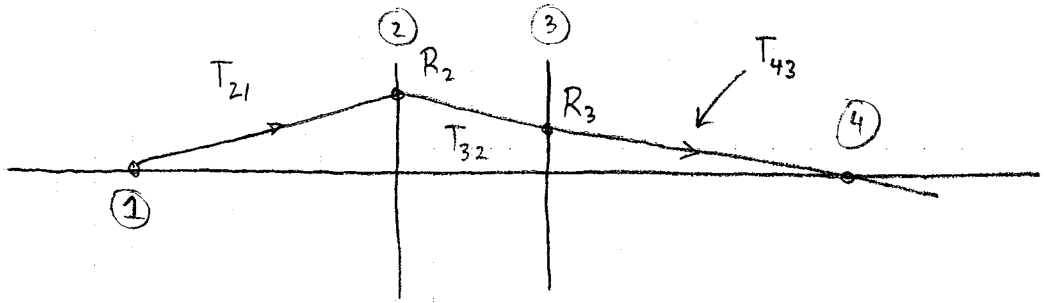
The final image is located at

$$\frac{1}{s_{i_2}} = \frac{1}{f_2} - \frac{1}{s_{o_2}} = -\frac{1}{20cm} + \frac{1}{7.5cm} = \frac{1}{12\,cm}.$$

$\Rightarrow$ $S_i = 12\,cm$ to the right of the second lens.



2. The following diagram represents the thin lens system:



The transfer matrix T_{21} is

$$T_{21} = \begin{pmatrix} 1 & 0 \\ s_o & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 50\,\text{cm} & 1 \end{pmatrix}$$

The first refraction matrix for the thin lens is

$$R_2 = \begin{pmatrix} 1 & -1/f_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1/10\,\text{cm} \\ 0 & 1 \end{pmatrix}$$

The transfer matrix T_{32} is

$$T_{32} = \begin{pmatrix} 1 & 0 \\ 5\,\text{cm} & 1 \end{pmatrix}$$

The refraction matrix for the second thin lens is

$$R_3 = \begin{pmatrix} 1 & -1/f_2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1/20\,\text{cm} \\ 0 & 1 \end{pmatrix}$$

The last transfer matrix is

$$T_{43} = \begin{pmatrix} 1 & 0 \\ s_i & 1 \end{pmatrix}$$

Use Mathematica or something similar.

The system matrix is

$$S = T_{43} R_3 T_{32} R_2 T_{21} = \begin{pmatrix} -5/2 & -3/40\,\text{cm} \\ 30\,\text{cm} - \frac{5}{2} s_i & \frac{1}{2} - \frac{3}{40\,\text{cm}} \cdot s_i \end{pmatrix}$$

When the initial state is $y_i = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$ the final state is

$$y_f = S y_i = \begin{pmatrix} -5\alpha/2 \\ \left(30\,\text{cm} - \frac{5 s_i}{2}\right) \alpha \end{pmatrix}$$

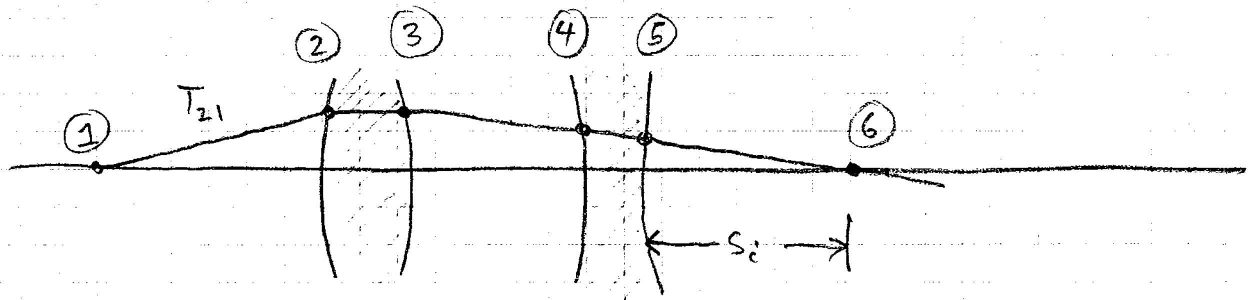
If the image is formed when $y_f = \begin{pmatrix} \alpha' \\ 0 \end{pmatrix}$ then $\left(30\,\text{cm} - \frac{5 s_i}{2}\right) = 0$

$$\Longrightarrow s_i = \frac{60\,\text{cm}}{5} = 12\,\text{cm}.$$

3. Now we consider each lens to be 1 cm thick but located at the same positions as in the previous problems. Thus, for example, the first transfer matris is

$$T_{21} = \begin{pmatrix} 1 & 0 \\ 49.5\,\text{cm} & 1 \end{pmatrix}$$

The optical system now looks like this:



The refraction matrix for the first spherical surface is

$$R_2 = \begin{pmatrix} 1 & -D_2 \\ 0 & 1 \end{pmatrix}$$

where $D_2 = \dfrac{(n_e - 1)}{R_1} = \dfrac{1.5 - 1}{10\,\text{cm}} = \dfrac{1}{20\,\text{cm}}$

Likewise, $R_3 = \begin{pmatrix} 1 & -D_3 \\ 0 & 1 \end{pmatrix}$

where $D_3 = \dfrac{(1 - n_e)}{-R_1} = \dfrac{1 - 1.5}{-10\,\text{cm}} = \dfrac{1}{20\text{cm}}$

Similarly, $R_4 = \begin{pmatrix} 1 & -D_4 \\ 0 & 1 \end{pmatrix}$ where $D_4 = \dfrac{n_e - 1}{R_2} = -\dfrac{1}{40\text{cm}}$

and $R_5 = \begin{pmatrix} 1 & -D_5 \\ 0 & 1 \end{pmatrix}$ where $D_5 = \dfrac{1 - n_e}{-R_2} = -\dfrac{1}{40\text{cm}}$

The transfer matrices are

$$T_{21} = \begin{pmatrix} 1 & 0 \\ 49.5\text{cm} & 1 \end{pmatrix}, \quad T_{32} = \begin{pmatrix} 1 & 0 \\ 1\text{cm}/1.5 & 1 \end{pmatrix}$$

$$T_{43} = \begin{pmatrix} 1 & 0 \\ 4\,\text{cm} & 1 \end{pmatrix}, \quad T_{54} = \begin{pmatrix} 1 & 0 \\ 1\text{cm}/1.5 & 1 \end{pmatrix}$$

$$T_{65} = \begin{pmatrix} 1 & 0 \\ s_i & 1 \end{pmatrix}$$

The system matrix is

$$S = T_{65} R_5 T_{54} R_4 T_{43} R_3 T_{32} R_2 T_{21}$$

When the initial state is

$$y_i = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$$

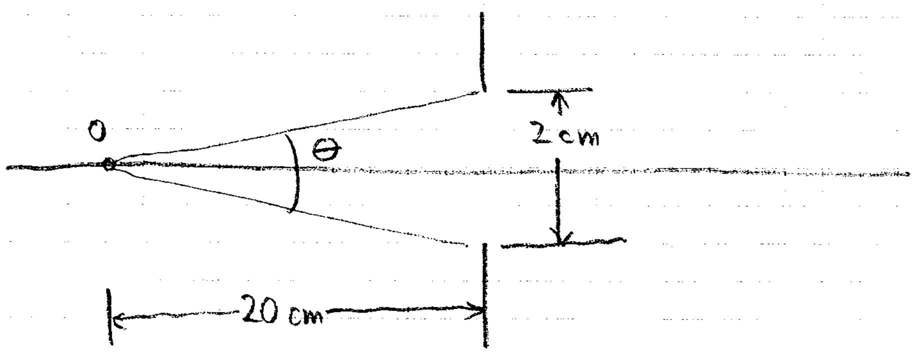
the final state is

Use Mathematica or something similar.

$$y_f = S y_i = \begin{pmatrix} -2.306\,\alpha \\ (30.861\text{ cm} - 2.306\, s_i)\,\alpha \end{pmatrix} = \begin{pmatrix} \alpha' \\ 0 \end{pmatrix}$$

Therefore, $s_i = \dfrac{30.861\text{ cm}}{2.306} = 13.38\text{ cm}$.

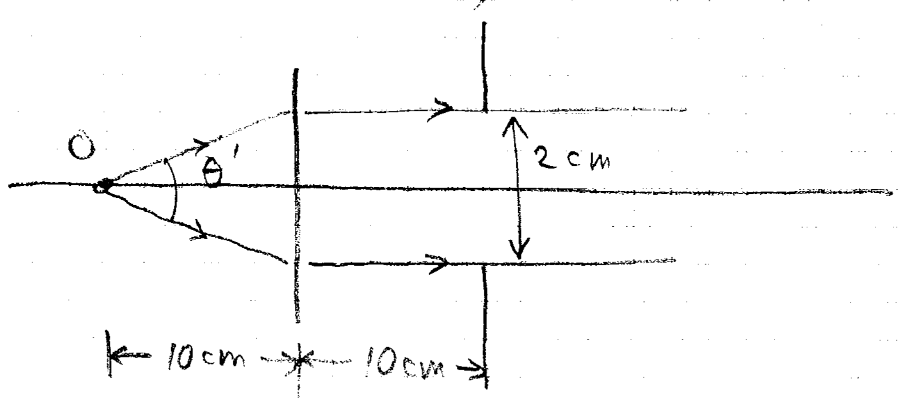
4. (a) With no optical elements between the object and the aperture stop, the entrance pupil is just the angle subtended by the aperture stop:



Thus, $\theta = \frac{2\text{ cm}}{20\text{ cm}} = 0.1\text{ rad} = 5.73^\circ$.

(b) If a thin lens with focal length $f = 10$ cm is placed mid-way between the object and the aperture stop then the object will be at the focal point.

The marginal ray now looks like this:



The angular extent of the entrance pupil is now

$$\theta' = \frac{2\text{cm}}{10\text{cm}} = 0.2\text{ rad} = 11.46^\circ .$$