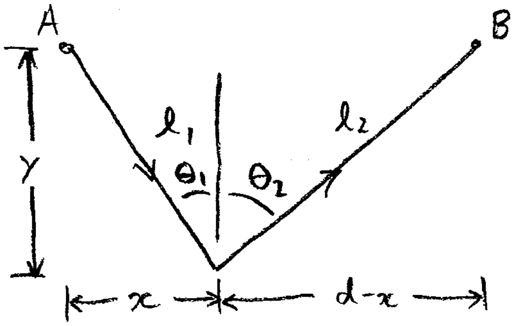
Assignment #5

1.



The time it takes for the light to travel from point A to point B is

$$t = \frac{l_1}{c} + \frac{l_2}{c} = \frac{\sqrt{y^2+x^2}}{c} + \frac{\sqrt{y^2+(d-x)^2}}{c}$$

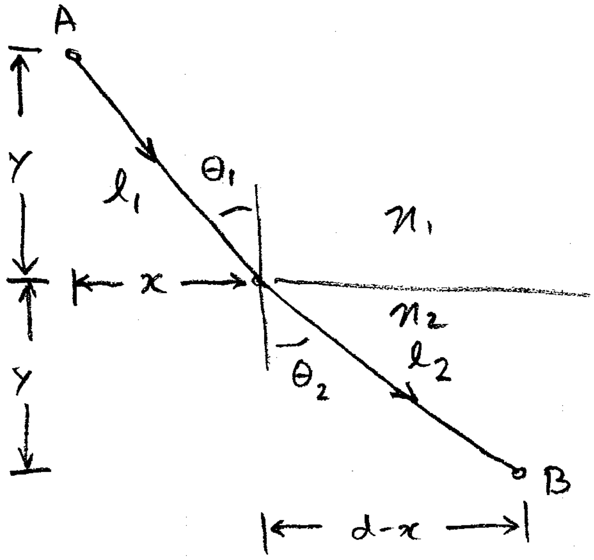
If this is stationary with respect to variations in x, then

$$\frac{dt}{dx} = \frac{x}{l_1 c} - \frac{(d-x)}{l_2 c} = 0$$

But $\frac{x}{l_1} = \sin\theta_1$ and $\frac{(d-x)}{l_2} = \sin\theta_2$

Therefore, $\sin\theta_1 = \sin\theta_2$.

2.



The time it takes for light to travel from point A to point B is

$$t = \frac{n_1 l_1}{c} + \frac{n_2 l_2}{c} = \frac{n_1}{c}\sqrt{x^2+y^2} + \frac{n_2}{c}\sqrt{(d-x)^2+y^2}$$

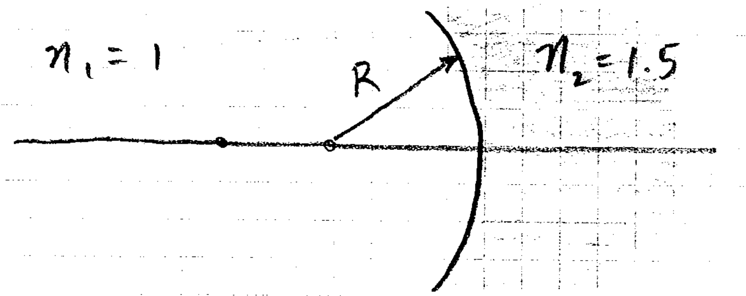
If this is stationary with respect to variations in x, then

$$\frac{dt}{dx} = \frac{n_1}{c}\frac{x}{l_1} - \frac{n_2}{c}\frac{(d-x)}{l_2} = 0$$

But $\frac{x}{l_1} = \sin\theta_1$ and $\frac{d-x}{l_2} = \sin\theta_2$

Therefore $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

3.



For a single spherical refracting surface,

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} .$$

The object focal point, f_o, corresponds to s_o when $s_i \to \infty$.

Thus, $\dfrac{n_1}{f_o} = \dfrac{n_2 - n_1}{R} \Rightarrow f_o = R\dfrac{n_1}{n_2 - n_1}$

$$= \frac{(-10\,cm)(1)}{(1.5 - 1)}$$

$$= -20\ cm .$$

The image focal point, f_i, corresponds to s_i when $s_o \to \infty$.

Thus, $\dfrac{n_2}{f_i} = \dfrac{n_2 - n_1}{R} \Rightarrow f_i = \dfrac{R\,n_2}{n_2 - n_1} = \dfrac{(-10cm)(1.5)}{(1.5 - 1.0)}$

$$= -30\ cm$$

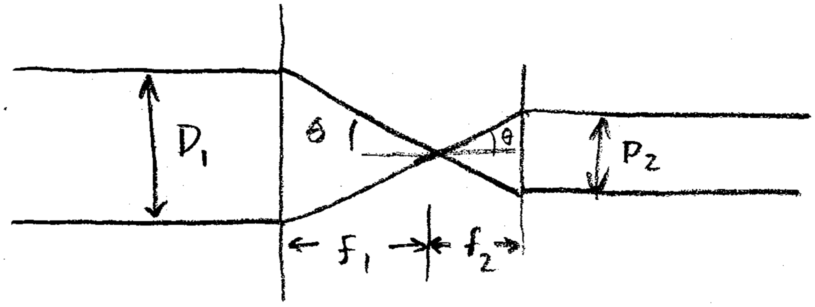
The fact that the object focal length is negative implies that parallel rays will not be produced by an object placed anywhere to the left of the interface.

(b) When air is replaced by water, $n_1 = 1.33$.

Then, $f_o = \dfrac{R\,n_1}{n_2 - n_1} = \dfrac{(-10\,cm)(1.33)}{1.5 - 1.33} = -78\ cm$

$$f_i = \frac{R\,n_2}{n_2 - n_1} = \frac{(-10\,cm)(1.5)}{1.5 - 1.33} = -88\ cm .$$

4.



$$\frac{D_1}{2} = f_1 \tan\theta \qquad \frac{D_1}{f_1} = \frac{D_2}{f_2}$$

$$\frac{D_2}{2} = f_2 \tan\theta$$

$$\Rightarrow \quad D_2 = D_1 \frac{f_2}{f_1} \quad .$$

The intensity varies as $\frac{D_1^2}{D_2^2}$ so the final intensity is

$$I = I_0 \left(\frac{f_1}{f_2}\right)^2 \quad .$$

Recall that intensity is defined as power per unit area. If the area decreases but the power remains the same then the intensity increases.