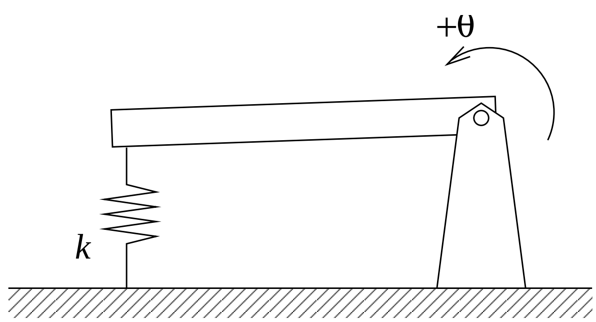
Physics 422 - Spring 2015 - Midterm Exam, March 12$^{\text{th}}$

Answer all questions in the exam booklets provided.
There are 6 questions - please answer **any four** *of them.*
Explain your reasoning clearly but concisely.
(but it is not expected to be at the same level as the homework)
Clearly indicate which work is to be graded.
Each question is of equal weight.
You can use one page of your own notes/formulas.

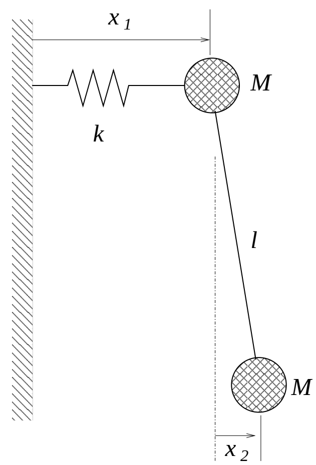
1. A heavy beam, with length L and mass M, pivots about a fixed axis at one end and bounces on a spring with spring constant k at the other end as shown:



(a) When in equilibrium, the beam is horizontal. Write the differential equation that describes the angle θ measured with respect to the equilibrium position with $+\theta$ in the direction shown on the figure. The moment of inertia of the beam about the fixed axis is $I = \frac{1}{3}ML^2$.

(b) What is the angular frequency, ω_0, of small oscillations about the equilibirum position?

2. One mass, M, is constrained to move only in the horizontal direction and is attached to a wall by a spring with spring constant k. A second mass, M, hangs from the first mass on a string of length ℓ as shown:



The top mass has displacement x_1, measured with respect to its equilibrium position, and the bottom mass has displacement x_2, measured with respect to the top mass.

(a) Write the equations of motion for $x_1(t)$ and $x_2(t)$. *Remember that Newton's second law only applies in an inertial reference frame.*

(b) Show that the angular frequencies of the normal modes of oscillation are

$$\omega^2 = \frac{2g/\ell + \omega_0^2 \pm \sqrt{4g^2/\ell^2 + \omega_0^4}}{2}$$

where $\omega_0^2 = k/M$.

3. Consider a string with tension T on which N beads with equal mass m and spacing ℓ are attached. If all the beads are motionless at $t = 0$ then the motion of an arbitrary bead k can be described by a sum over normal modes of oscillation:

$$y_k(t) = \sum_{n=1}^{N} a_n \sin\left(\frac{nk\pi}{N+1}\right)\cos(\omega_n t)$$

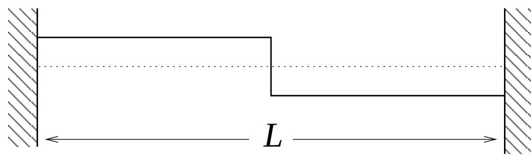
where

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right).$$

Find an expression for a_n that will solve the initial value problem where all masses are initially in their equilibrium position except for bead m, which is displaced by an amplitude A at $t = 0$.
It may be useful to recall that

$$\sum_{k=1}^{N} \sin\left(\frac{nk\pi}{N+1}\right)\sin\left(\frac{mk\pi}{N+1}\right) = \frac{N}{2}\delta_{nm}$$

4. A string of length L is deformed into the shape shown below, where the step occurs in the center, and is released from rest:



If the general form of the solution can be written

$$y(x,t) = \sum_n a_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)$$

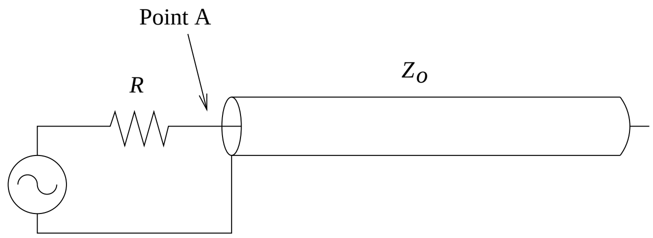
show that in order to satisfy the initial conditions, we must have $a_n = 0$ whenever n is an odd integer.

5. The current $i(t)$ in a series RLC circuit must satisfy the equation:

$$L\frac{di}{dt} + i(t)R + \frac{1}{C}\left(Q_0 + \int_0^t i(t)dt\right) = 0$$

where Q_0 is the initial charge on the capacitor. Find an expression for R that will produce critical damping.

6. **(a)** An ideal voltage source is attached via a resistor to a coaxial cable of length L as shown:



The far end of the cable is an open circuit and the source and cable impedances are matched so that $R = Z_0$. If signals propagate in the cable with a speed v, what is the lowest frequency that will result in a maximum voltage, measured at point A?

(b) What would be the lowest frequency resulting in a maximum voltage measured at point A if the far end of the cable was shorted?

It may help to sketch the incident and reflected waves in the cable.