Physics 422 - Spring 2015 - Assignment #4
Due Friday, March 6^{th}

1. The Euler-Bernoulli equation can be used to describe the time-dependent deflection of a beam on which no external forces act:

$$YI\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial^2 w}{\partial t^2} \quad = \quad 0$$

In this equation, Y is the elastic modulus of the material of which the beam is made, μ is the linear mass density of the beam, and the term I is the area moment of intertia of the beam.

(a) Suppose a beam of length L is fixed at both ends so that $w(0,t) = w(L,t) = 0$. If we propose that a solution to the beam equation might be of the form $w(x,t) = A\sin(k_n x)e^{i\omega_n t}$, determine the discrete values k_n that will satisfy the boundary conditions.

(b) Determine the frequencies ω_n that will make the proposed solution satisfy the beam equation for a given choice of k_n.

(c) A wire bond of length $L = 1$ mm is used to make an electrical connection between integrated circuits using thin aluminum wire with a circular cross section and a diameter of 25 μm. If the elastic modulus of aluminum is $Y = 69 \times 10^9$ N/m^2 and its density is $\rho = 2.7 \times 10^6$ kg/m^3, calculate the lowest frequency (in Hz) at which the wire bond will vibrate. The area moment of inertia for a circle of radius r is $I = \pi r^4/2$.

2. A string of length L has mass density μ and tension T and is held fixed at both ends $x = 0$ and $x = L$. At time $t = 0$, the string is in its equilibrium position, *ie.* $y(x,0) = 0$, but has an initial velocity given by

$$\dot{y}(x,0) = vx(L-x). \tag{1}$$

Find an expression for the displacement of the string as a function of time for $t > 0$.

3. *(French, 7-9)* A symmetrical triangular pulse of maximum height 0.4 m and total length 1.0 m is moving in the positive x direction on a string on which the wave speed is 24 m/s. At $t = 0$ the pulse is entirely located between $x = 0$ and $x = 1$ m. Draw a graph of the transverse velocity versus time at $x = 1$ m.