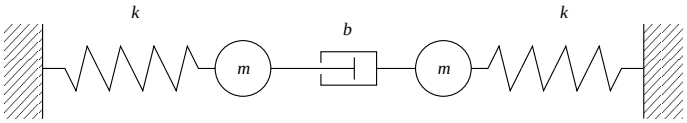
**Physics 422 - Spring 2015 - Assignment #3, Due Wednesday, February 18$^{\text{th}}$**

**1.** Consider a system of two springs and two masses that are connected together by a damper as shown:




The damper produces a force that is proportional to the relative velocity between the two ends. That is, if $x_1$ represents the position of the mass on the left, and $x_2$ represents the position of the mass on the right, then the damper will exert an equal and opposite force with *magnitude* $b(\dot{x}_2 - \dot{x}_1)$ on each mass. Furthermore, assume that the damping is weak, so that $\gamma = b/m < \omega_0 = \sqrt{k/m}$.

**(a)** Determine the net force that acts on each mass and write the set of coupled differential equations for $x_1$ and $x_2$, which you can assume are defined with respect to their equilibrium positions.

**(b)** Assume that a solution is of the form $x_i = A_i e^{\alpha t}$, substitute it into the set of differential equations and write the resulting system of algebraic equations in matrix form, $\mathbf{Ax} = \mathbf{0}$.

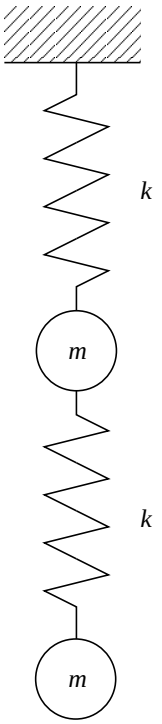
**(c)** Write an expression for the characteristic polynomial of which $\alpha$ must be a root in order to satisfy the matrix equation in part (b).

**(d)** Describe why you would expect $\alpha = \pm i\omega_0$ to be two of the roots of the characteristic polynomial? What kind of motion would this correspond to?

**(e)** Use polynomial division to factor $(\alpha^2 + \omega_0^2)$ out of the characteristic polynomial and use the quadratic formula to find expressions for the other two roots.

**(f)** Calculate the eigenvectors corresponding to the roots of the polynomial found in (d) and (e) and describe the resulting motion.

**2.** Consider a system of two identical masses hanging from two identical springs as shown:




Solve for the motion of each mass about their equilibrium positions when both masses are initially at rest but the bottom one is pulled down by a force $F$ and then released at $t = 0$.

Completing this problem will require several steps. Follow the examples discussed in class and in the text and be sure to explain clearly each step in your analysis.