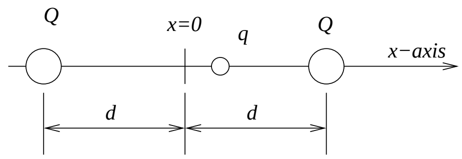
Physics 422 - Spring 2015 - Assignment #2, Due February 9$^{\text{th}}$

1. (a) A charge q with mass m is placed on the x-axis, in between two fixed charges Q as shown:



If the charge is constrained to move only along the x-axis, calculate the frequency of small oscillations about the equilibrium position.

(b) Sketch the potential energy $V(x)$ of the charge q for $-d < x < d$.

(c) Sketch the phase diagram, *ie.* curves in the $\dot{x}$-x plane, that describe the motion of the mass for both small amplitude oscillations $|x| \ll d$ and large amplitude oscillations $|x| \lesssim d$.

2. From the expression for the amplitude of a forced harmonic oscillator,

$$A(\omega) \quad = \quad \frac{F_0/m}{\sqrt{((\omega_0^2 - \omega^2)^2 + (\omega\omega_0)^2/Q^2}},$$

show that the maximum amplitude occurs at a frequency

$$\omega_m \quad = \quad \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$

and that the maximum amplitude is

$$A_m \quad = \quad \frac{F_0}{k}\frac{Q}{\sqrt{1 - 1/2Q^2}}$$

3. Consider a pendulum consisting of a mass m attached to a string of length ℓ. If the pendulum was initially in motion, but the *amplitude* of small oscillations is reduced by a factor of $1/e$ in time T, calculate the value of γ and Q for this oscillator.

4. Suppose that the support from which the pendulum in Qustion 3 was hung is moved in the horizontal direction with a displacement given by

$$x(t) \quad = \quad d \cos \omega t.$$

Calculate the maximum angle of steady state oscillations as a function of the driving frequency, ω.