Physics 422 - Spring 2015 - Assignment #1, Due February 2$^{\text{nd}}$

1. Consider the polynomial

$$Ax^2 + Bx + C \quad = \quad 0.$$

(a) What are the roots of the polynomial?
(b) What conditions must A, B, and C satisfy for the roots to be real?

2. Show that the complex valued function

$$z(t) = ae^{i\alpha}e^{i\omega t} + be^{i\beta}e^{i\omega t}$$

can be written in the form

$$z(t) = re^{i(\omega t + \varphi)}$$

and find expressions for r and φ in terms of the real numbers a, b, α and β.

3. Consider two springs with spring constants k_1 and k_2.
(a) Show that the effective spring constant, k_p, that would result if the springs were connected in parallel, is given by

$$k_p \quad = \quad k_1 + k_2.$$

(b) Show that the effective spring constant, k_s, that would result if the springs were connected in series, is given by

$$k_s \quad = \quad \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}.$$

4. Consider an object made out of elastic material of length L that has a uniform elastic modulus, Y, and a cross sectional area given by the function $A(\ell)$, where $0 \leq \ell \leq L$. Find an expression for the spring constant of the object.

5. A mass, m, is attached to one end of a spring with spring constant k and equilibrium length ℓ. The other end of the spring moves with constant velocity so that its position $X(t)$ at time t is given by

$$X(t) \quad = \quad \ell + vt.$$

Find an expression for the position of the mass as a function of time, $x(t)$, if at time $t = 0$ the mass is initially located at $x(0) = 0$ and is initially at rest, $\dot{x}(0) = 0$.