Assignment #7

1. The electric field at the screen is the vector sum of the electric field from light that passed through each slit.

The intensity on the screen is

$$I = v\varepsilon \langle |\vec{E}_1 + \vec{E}_2|^2 \rangle_T$$

where the average is taken over a suitable time interval.

When $|\vec{E}_1| = |\vec{E}_2|$ this is

$$I = 2I_0 \pm 2I_0 \cos\delta = 4I_0 \cos^2\frac{\delta}{2}$$

where δ is the average phase difference between the two electric fields.

When $|r_1 - r_2| < 3\lambda$, the two paths are coherent and $\delta = k(r_1 - r_2)$. Otherwise, δ is random and $\cos\delta$ averages to zero.

At a point y on the screen the path length difference is ay/s and the phase difference would be

$$\delta = \frac{2\pi a y}{\lambda s}$$

This will be coherent when $y < 3\left(\frac{\lambda s}{a}\right)$ and incoherent otherwise. Thus, the intensity will be

$$I(y) = \begin{cases} 4I_0 \cos^2\left(\frac{\pi a y}{\lambda s}\right) & \text{when } y < 3\lambda s/a \\ 2I_0 & \text{otherwise.} \end{cases}$$

2. The glass has a lower index of refraction than the film so there is a phase reversal in the light reflected from the top surface but not the bottom surface of the film.

Constructive interference occurs when

$$2nd = \left(m + \tfrac{1}{2}\right)\lambda$$

where d is the film thickness, n is the index of refraction of the film and λ is the wavelength in free space.

We do not necessarily know m but for constructive interference with orange light ($\lambda = 600$ nm) the possible values of thickness are $d = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{2n}$

m	d (μm)
9	1.676
10	1.853
11	2.029

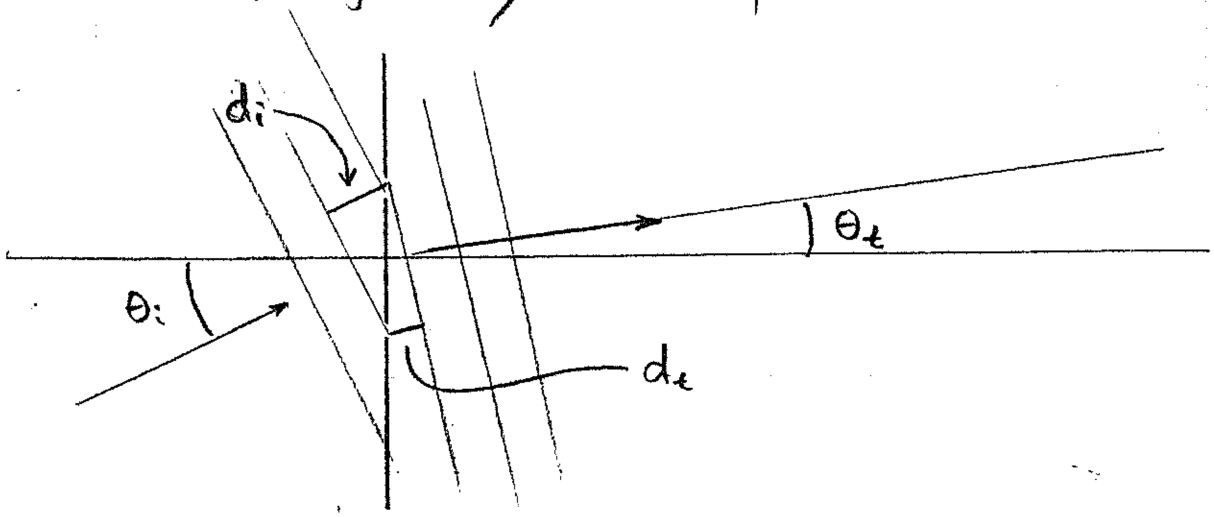
while for green light the possible thicknes is

m'	d' (μm)
11	1.691
12	1.838
13	1.985

If the nominal film thickness is 1.8 μm then most likely $m = 10$ and $m' = 12$ and the variation in the film thickness would be $1.853 - 1.838 = 0.015 \mu m$

The surface is flat at the level of 15 nm.

3. Consider the geometry of the problem below:



The phase difference of the incident light is

$$\delta_i = \frac{2\pi d_i}{\lambda} = \frac{2\pi a \sin\theta_i}{\lambda}$$

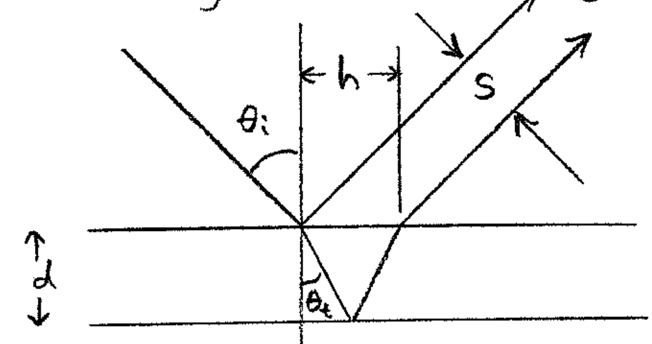
and the phase difference between the transmitted rays is

$$\delta_t = \frac{2\pi d_t}{\lambda} = \frac{2\pi a}{\lambda} \sin\theta_t$$

Constructive interference occurs when $\delta_t - \delta_i = 2m\pi$

Hence, $\sin\theta_t = \sin\theta_i + \frac{m\lambda}{a}$

4.(a) The spacing between the paths can be analyzed by considering the following geometry:



From Snell's law, $\sin\theta_i = n\sin\theta_t$ but $\theta_i = \pi/4$
so

$$\sin\theta_t = \frac{1}{n\sqrt{2}}$$

The horizontal displacement, h, as shown, is

$$h = 2d\tan\theta_t = \frac{2d\sin\theta_t}{\sqrt{1-\sin^2\theta_t}} = \frac{2d}{\sqrt{\frac{1}{\sin^2\theta_t}-1}}$$

$$= \frac{2d}{\sqrt{2n^2-1}}$$

When $d = 10$ mm and $n = 1.5$, this is

$$h = \frac{2(10\text{ mm})}{\sqrt{2(1.5^2)-1}} = 10.69\text{ mm}.$$

However, the lateral separation between the two beams is

$$s = \frac{h}{\sqrt{2}} = 7.56\text{ mm}.$$

(b) The difference in optical path lengths is

$$(n_1 L - n_2 L) = m\lambda \quad \text{for constructive interference.}$$

When m changes by 50, n_2 has changed by the amount

$$\Delta n_2 = \frac{50\,(500\text{ nm})}{5\text{ cm}} = 0.0005 .$$

5. A Fabry-Perot interferometer has an amplitude reflection coefficient of $r = 0.8944$.

(a) The coefficient of finesse is the term F in the expression

$$\frac{I_t}{I_i} = \frac{1}{1 + F\sin^2 \delta/2}$$

and is written in terms of r as

$$F = \left(\frac{2r}{1-r^2}\right)^2 = \left(\frac{2(0.8944)}{1-(0.8944)^2}\right)^2 = 80.$$

(b) The half-width of a fringe is given by

$$\gamma = 4\sin^{-1}\left(\frac{1}{\sqrt{F}}\right) \approx \frac{4}{\sqrt{F}} = 0.447.$$

(c) The Finesse is $\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi}{2}\sqrt{F} = 14$.

(d) The contrast factor is defined

$$C = \frac{(I_t/I_i)_{max}}{(I_t/I_i)_{min}}$$

but since $\frac{I_t}{I_i} = \frac{1}{1 + F\sin^2\delta/2}$ the maximum value occurs when $\sin^2\delta/2 = 0$ and the minimum occurs when $\sin^2\delta/2 = 1$.

Thus, $C = 1 + F = 81$.