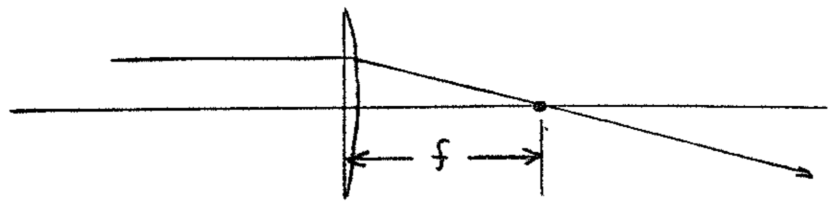
# Assignment #5



1. Focal length of a planar-convex lens in air:



Let $R_1$ be the radius of curvature of the front surface and $R_2$ be the radius of curvature of the back surface. The thin lens equation is

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When $R_1 \to \infty$, $R_2 \to -50$ mm, $s_o \to \infty$ the image is formed at the focal point, $s_i = f$.

Thus, $\frac{1}{f} = (n_e - 1)\left(-\frac{1}{R_2}\right)$

(a) $$f = \frac{-R_2}{n-1} = \frac{50 \text{ mm}}{(1.5-1)} = \frac{50 \text{ mm}}{0.5} = 100 \text{ mm}.$$

(b) If the lens was in water then the thin lens equation becomes

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{n_e - n_w}{n_w}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = \frac{n_e - n_w}{n_w}\left(\frac{-1}{R_2}\right), \quad f = \frac{-R_2 n_w}{n_e - n_w} = \frac{(50\text{mm})(1.33)}{1.5 - 1.33}$$

$$= 391 \text{ mm}.$$

2. For a biconcave lens, the thin lens equation can be written

$$\frac{1}{s_o} + \frac{1}{s_i} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

In this case, we are give the object position,

$$s_o = 8 \text{ cm}$$

the index of refraction, $n = 1.5$, and the radii of curvature:

$$R_1 = -20 \text{ cm}$$
$$R_2 = +10 \text{ cm} .$$

The image distance is then

$$s_i = \left( (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{1}{s_o} \right)^{-1}$$

$$= \left( (1.5-1)\left(\frac{1}{-20\text{ cm}} - \frac{1}{10\text{ cm}}\right) - \frac{1}{8\text{ cm}} \right)^{-1}$$

$= -5$ cm, measured from the vertex of the lens.

The transverse magnification is

$$M_T = -\frac{s_i}{s_o} = \frac{5\text{cm}}{8\text{ cm}} = 5/8 = 0.625$$

The image is reduced in size but is upright.
The image is a virtual image.
If the object has a height of 1 cm, then the image will have a height of 6.25 mm.

When the thickness of the lens is taken into consideration, the thick lens equation can be used to calculate the focal length:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)$$

$$= (1.5-1)\left(\frac{1}{-20\text{ cm}} - \frac{1}{10\text{ cm}} + \frac{(1.5-1)(5\text{ cm})}{(1.5)(-20\text{ cm})(10\text{ cm})}\right)$$

So $f = -12.632$ cm.

The principal planes are located at distances

$$h_1 = -\frac{f(n-1)d}{nR_2} = -\frac{(-12.632\text{ cm})(1.5-1)(5\text{ cm})}{(1.5)(10\text{ cm})}$$

$$= 2.105\text{ cm}$$

$$h_2 = \frac{-f(n-1)d}{nR_1} = \frac{-(-12.632\text{ cm})(1.5-1)(5\text{ cm})}{(1.5)(-20\text{ cm})}$$

$$= -1.053\text{ cm}$$

The position of the object, measured with respect to the first principal plane, is

$$s_o' = s_o + h_1 = 8\text{ cm} + 2.105\text{ cm} = 10.105\text{ cm}$$

The image position, measured with respect to the second principal plane is obtained from

$$\frac{1}{s_i'} + \frac{1}{s_o'} = \frac{1}{f}$$

So. $$s_i' = \left(\frac{1}{f} - \frac{1}{s_o'}\right)^{-1} = \left(\frac{1}{-12.632\text{cm}} - \frac{1}{10.105\text{ cm}}\right)^{-1} = -5.614\text{ cm}$$

The image position measured with respect to the second vertex is

$$s_i = s_i' + h_2$$

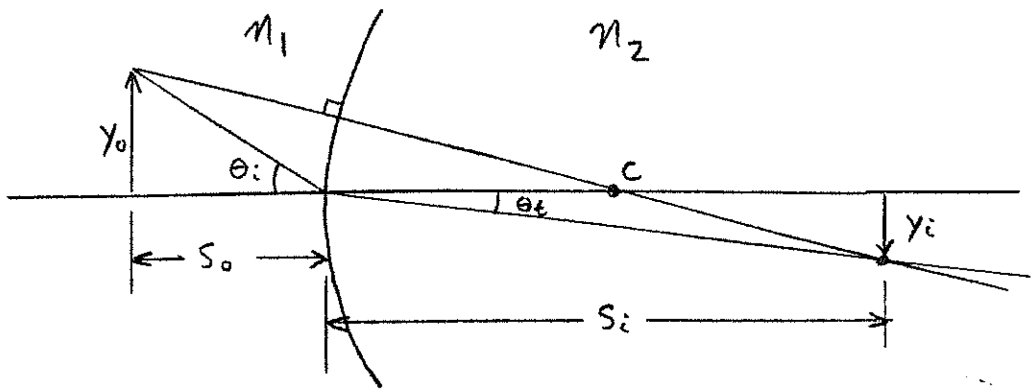
$$= -5.614\text{ cm} + (-1.053\text{ cm})$$

$$= -6.667\text{ cm}$$

The transverse magnification is

$$M_T = -\frac{s_i'}{s_o'} = -\frac{-5.614\text{ cm}}{10.105\text{ cm}} = 0.556$$

3. The following diagram shows the geometry for a single refracting surface




From this diagram we see that $y_o = s_o \tan\theta_i$ and $y_i = s_i \tan\theta_t$, but in the small angle approximation, $\tan\theta \approx \theta$.

Thus, $M_T = \dfrac{y_i}{y_o} = -\dfrac{s_i\,\theta_t}{s_o\,\theta_i}$

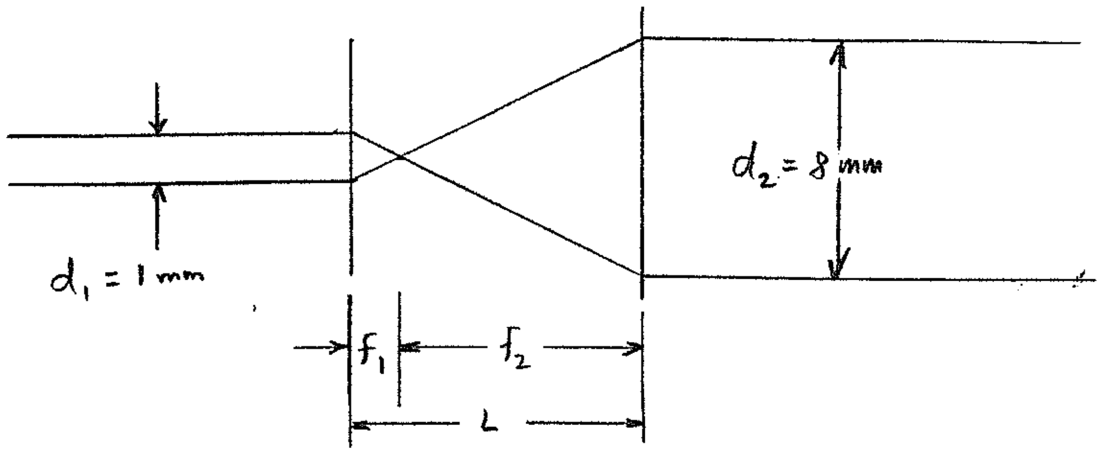
However, from Snell's law, $n_1\theta_i = n_2\theta_t$

So $\theta_t/\theta_i = n_1/n_2$.

Therefore, $M_T = \dfrac{-n_1 s_i}{n_2 s_o}$

4. The parallel rays of the laser beams correspond to object and image distances at infinity.

The geometry of the lens system is as follows:




There are similar triangles so $\frac{d_1}{f_1} = \frac{d_2}{f_2}$.

Thus, $f_2 = \frac{d_2}{d_1} f_1 = \frac{(8 \text{ mm})}{(1 \text{ mm})}(50.0 \text{ mm})$

$= 400$ mm.

The separation between the lenses must be

$\ell = f_1 + f_2 = (50 \text{ mm}) + (400 \text{ mm}) = 450 \text{ mm}$