Physics 422 - Spring 2014 - Midterm Exam, March 13th

1. A heavy beam, with length L and moment of inertia I, is constrained to rotate about its center of mass, but is connected to the ground by springs at each end as shown:
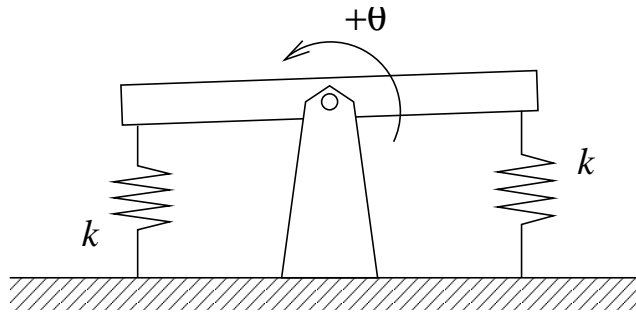

(a) What is the torque acting on the beam when it is rotated by a small angle θ in the direction shown?

(b) Write the differential equation that describes the angle, $\theta(t)$.

(c) What is the angular frequency of free oscillations, assuming there is no damping.

2. A mass m is placed in the center of a trampoline and is observed to oscillate with period of 1 second. It takes 5 seconds for the the amplitude of oscillations to be reduced to $1/2$ of the initial amplitude.

(a) What is the Q value for this oscillating system?

(b) During an earthquake, the mass is observed to be oscillating with a period of 1 second and an amplitude of 10 cm. What is the amplitude of the seismic waves on the surface of the earth that are moving the trampoline?

3. Two masses are hung from strings of equal length ℓ and are attached together by a spring with spring constant, k as shown:
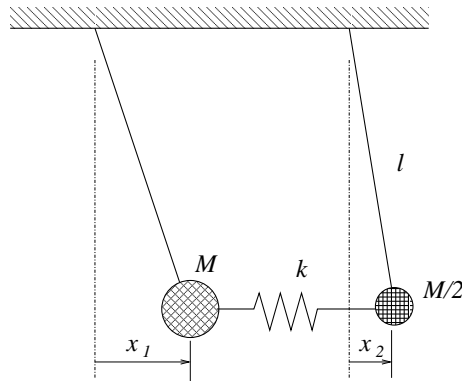
The mass on the right, with displacement x_2, is half the mass of the one on the left, which has displacement x_1.

(a) Write the equations of motion for $x_1(t)$ and $x_2(t)$.

(b) Show that the angular frequencies of the normal modes of oscillation are

$$\omega_1 = \sqrt{g/\ell}$$
$$\omega_2 = \sqrt{3k/m + g/\ell}.$$

It might be useful to analyze this problem in terms of the variables $\omega_0^2 = k/m$ and $\omega_g^2 = g/\ell$.

4. Two identical masses are hanging by springs with different spring constants as shown:
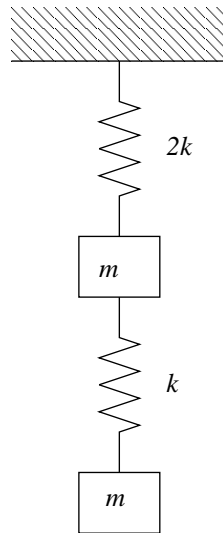
(a) Write the equations of motion for the two masses using suitably chosen coordinates in an inertial reference frame.

(b) Calculate the frequencies of the two normal modes of oscillation in terms of the parameter $\omega_0 = \sqrt{k/m}$.

5. A guitar string of length L, tension T and mass per unit length μ is initially plucked into a triangular shape as shown in the diagram below:
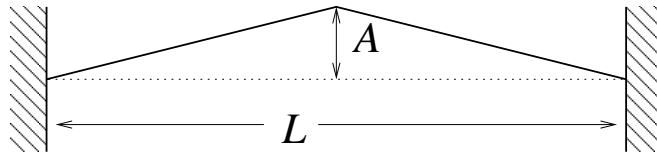
The initial shape of the string can be written in the form

$$y(x) \;=\; \sum_n a_n \sin\left(\frac{n\pi x}{L}\right)$$

where

$$a_n \;=\; \begin{cases} \pm\frac{8A}{n^2\pi^2} & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases}$$

(a) What is the lowest frequency that will be excited when the string is released from rest?

(b) If, after the string was released from rest, a finger was lightly touched to the point $x = \frac{1}{2}L$, constraining the solution to the wave equation to satisfy $y(\frac{1}{2}L, t) = 0$ in addition to the previous boundary conditions $y(0, t) = y(L, t) = 0$, would the string continue to vibrate and produce sound? Explain why or why not.

(c) If, after the string was released from rest, a finger was instead lightly touched to the point $x = \frac{2}{3}L$, constraining the solution to the wave equation to satisfy $y(\frac{2}{3}L, t) = 0$ in addition to the previous boundary conditions $y(0, t) = y(L, t) = 0$, would there be any sound produced? What would its dominant frequency be?

6. A signal generator is attached to a coaxial cable of length L in which signals propagate with velocity v and which has a characteristic impedance Z_0 as shown.
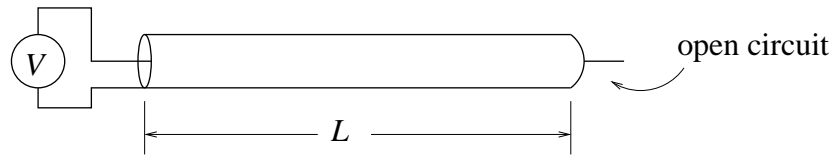
The far end of the cable is an open circuit and the source has an impedance much less than Z_0. If the voltage source produces a signal that is initially zero, but is switched on to produce a constant voltage V for all times $t > 0$, sketch the voltage as a function of time that would be observed at the open end of the cable for times $0 < t < 4L/v$.