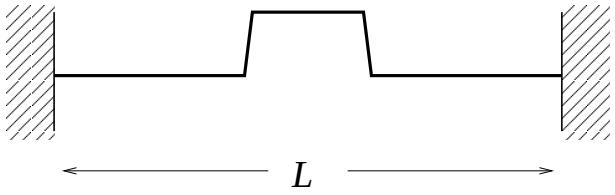
Physics 422 - Spring 2014 - Assignment #4, Due March 10th

1. A string of length $L = 8$ m, tension $T = 4$ N and linear mass density $\mu = 1$ kg/m is initially at rest, but has an initial displacement described by the function

$$f(x) = \begin{cases} 0 & x < 3L/8 \\ 1 & 3L/8 < x < 5L/8 \\ 0 & x > 5L/8 \end{cases}$$



If the ends of the string are fixed, so that they cannot move, sketch the shape of the string at times $t_1 = 1$ s, $t_2 = 3$ s and $t_3 = 4$ s.

2. A string of length L, which is fixed at both ends, has no initial displacement but has a transverse velocity described by the function

$$f(x) = ux(x - L)$$

where u is a constant. The general solution to the wave equation for the string,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

can be written

$$y(x,t) = \sum_n a_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t + \alpha_n)$$

where $\omega_n = n\pi v/L$ are the frequencies of the normal modes of oscillation. Find expressions for a_n and α_n that satisfy the initial conditions.

3. A pulse of length ℓ on a long string, with linear mass denisty μ, moves to the right with velocity v. At $x = 0$ the string is tied to a long rope which has linear mass density 4μ. If the pulse encounters the discontinuity at time $t = 0$, sketch the shape of the pulses on the string and the rope at time $t = 3\ell/v$.

4. A signal generator drives a voltage pulse with an amplitude of 1 V and width 10 ns on a coaxial cable of length $L = 30$ m which has characteristic impedance $Z = 50\ \Omega$ and a speed of signal propagation of 20 cm/ns. The other end of the cable is connected to an oscilloscope which graphs the voltage at that end of the cable as a function of time. Sketch the waveform that would be observed on the oscilloscope for the following combinations of the source impedance, Z_s, of the signal generator, and input impedance, Z_o of the oscilloscope:
(a) $Z_s = 0\ \Omega$, $Z_o = 50\ \Omega$
(b) $Z_s = 0\ \Omega$, $Z_o = 1\ \mathrm{M}\Omega$
(c) $Z_s = 50\ \Omega$, $Z_o = 50\ \Omega$
(d) $Z_s = 50\ \Omega$, $Z_o = 1\ \mathrm{M}\Omega$
(e) $Z_s = 100\ \Omega$, $Z_o = 1\ \mathrm{M}\Omega$