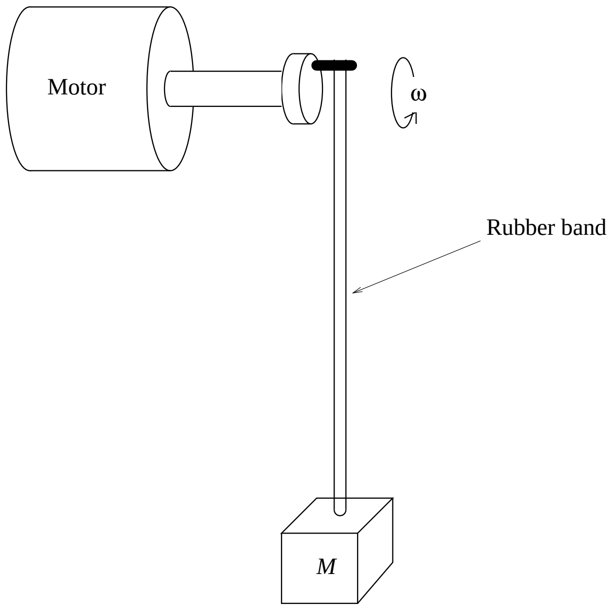
Physics 422 - Spring 2014 - Assignment #3, Due February 28$^{\text{th}}$

1. A log with mass $M = 100$ kg is floating in Lake Michigan, which contains fresh water, so its density is $\rho = 1$ g/cm^3. The log has a cross sectional area, $A = 500$ cm^2, and is oriented vertically as shown:

(a) If the log is subjected to waves with amplitude A and angular frequency ω, find an expression for the time-dependent driving force, $F(t)$, which acts on the log. Assume that one end of the log always remains out of the water.

(b) If the water exerted a damping force on the floating log such that $\gamma = 10^{-2}$ s^{-1} and waves were incident with a period $T = 6$ s, explain whether one should expect to observe large amplitude oscillations of the log?

2. A mass is attached to a rubber band with spring constant k which is suspended from a pin on a small wheel that is driven by a motor with angular frequency ω as shown:



(a) If the motor stops, the mass continues to oscillate but the ampitude decreases by a factor of $1/e$ in time T. Find an expression for the Q value of this oscillating system.

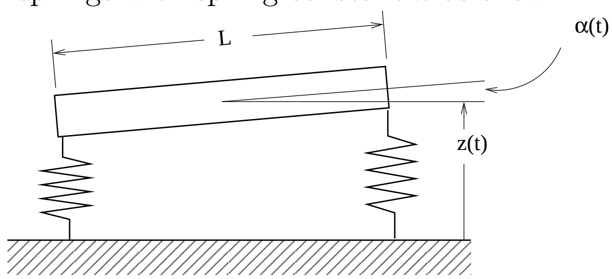
(b) If the pin on the wheel is at a radius r from the axis of the motor, find an expression for the vertical component of its position as a function of time when the motor rotates with angular frequency ω.

(c) If the length, x, of the rubber band is measured with respect to the vertical position of the pin, write a differential equation for $x(t)$. *Remember that $x(t)$ is defined in a non-intertial reference frame.*

(d) At what rotation frequency, ω, will the amplitude of steady-state oscillations be maximal? First estimate this in the limit where $\gamma \ll \omega_0$ and then calculate it exactly.

(e) How much should the frequency, ω, be increased or decreased to yield oscillations with amplitudes that are $1/2$ of the maximum amplitude?

3. A beam of length L with mass M and moment of inertia I is supported by two identical springs with spring constant k as shown:

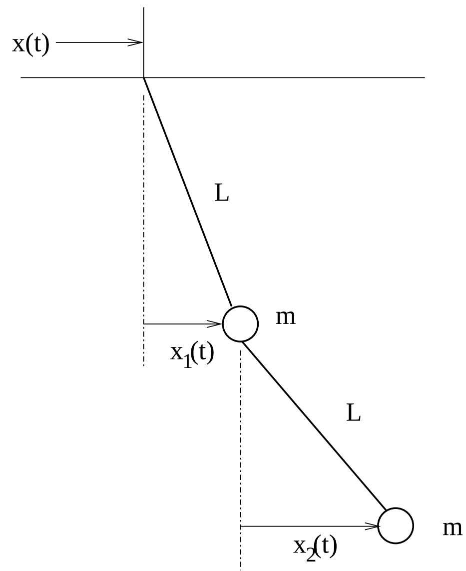


(a) If $z(t)$ represents the height of the center of mass and $\alpha(t)$ is the angle of the beam, write the coupled set of differential equations for $z(t)$ and $\alpha(t)$. Assume that $\alpha(t)$ is small enough that the approximation $\sin\alpha \approx \alpha$ is valid. Feel free to re-define $z(t)$ so that it is measured with respect to the equilibrium position instead of the floor. Just explain this clearly in your solution.

(b) Find the frequencies of the two normal modes of vibration.

(c) Solve the equations of motion when the beam is initially at rest, but one spring is initially compressed by a distance d, while the other spring remains in its equilibrium position.

4. A coupled pendulum consists of two identical masses attached to strings of length L as shown.



(a) The position of the top end of the string is driven at a frequency ω so that $x(t) = A\cos\omega t$. Calculate the frequencies ω that will produce resonances in the motion of the masses.

(b) Describe the motion (a qualitative description is fine) of the individual masses for each of the two resonant frequencies.