Physics 422 - Spring 2014 - Assignment #2, Due February 7th

1. A mass attached to a spring (which has neglibible mass) is described by the differential equation

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1}$$

which has a solution that can be written

$$x(t) = Ae^{-\gamma t/2}\cos(\omega t). \tag{2}$$

What is the total energy of the system (kinetic plus potential) as a function of time?

2. Suppose a mass m is subjected to a force that is described by the potential energy function

$$V(x) = \frac{b}{2x^2} - \frac{a}{x}. \tag{3}$$

You could calculate the force on the mass using $F = -dV/dx$, but you can also analyze this problem using energy considerations.
a. Sketch the shape of the function $V(x)$ and compare it to the shape of the potential energy function for a spring, $V(x) = \frac{1}{2}k(x - x_0)^2$.
b. Calculate the equilibrium position, x_0, for which the force vanishes.
c. Write an expression for the potential energy in terms of a new variable, $x' = x - x_0$.
d. Write the Taylor series expansion of $V(x')$ to second order in x'.
e. Express the total energy E in terms of the kinetic energy and the potential energy, keeping only terms to second order in x'.
e. Compare this expression with the total energy of a mass-spring system, $(E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2)$ and determine, by inspection, the angular frequency of small oscillations about the equilibrium position, x_0.

3a. Draw the phase space diagram (*i.e.*, curves of $\dot{x}$ *vs* x for different values of E) for a mass-spring system in which the spring exerts a force $F(x) = -k(x-x_0)$, where x_0 is the position of the mass when in a state of equilibrium.
b. Draw the phase space diagram for a mass subjected to a force described by the potential energy function

$$V(x) = \frac{b}{2x^2} - \frac{a}{x}. \tag{4}$$

Be sure to identify the cases where $E > 0$, $E = 0$ and $E < 0$.