Assignment # 9

1. The focal length of a thin lens is

$$\frac{1}{f_m} = \frac{1}{S_o} + \frac{1}{S_i} = \left(\frac{n_e}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n_m is the index of refraction of the medium and n_e is the index of refraction of the lens.

This can be written,

$$\frac{1}{f_m} = (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \left(\frac{n_e}{n_m} - n_e\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{n_e(n_m - 1)}{n_m(n_e - 1)}\,(n_e - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left[1 - \frac{n_e(n_m - 1)}{n_m(n_e - 1)}\right] \cdot \frac{1}{f_a}$$ or something equivalent.

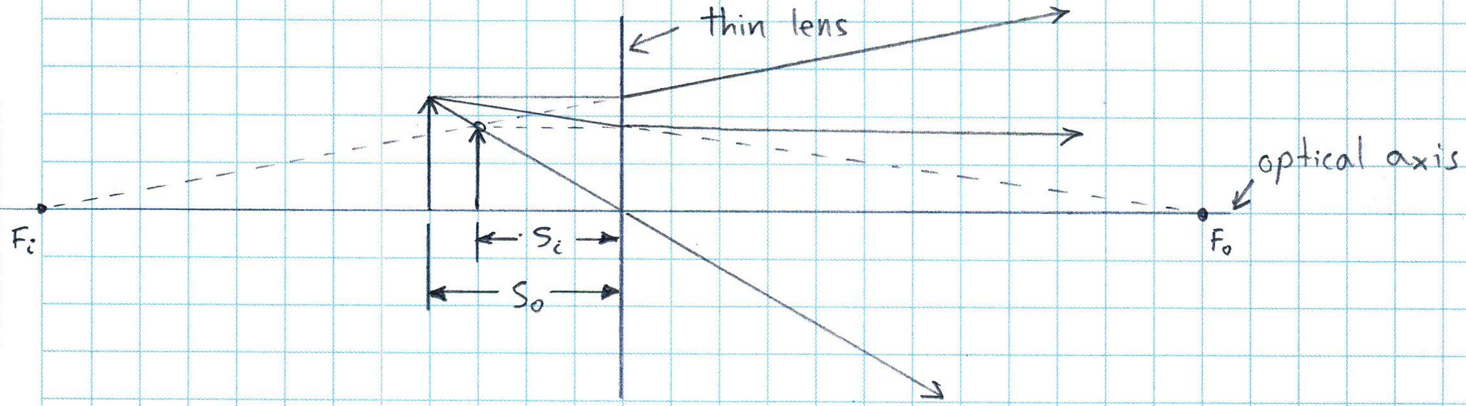
To calculate the focal length in water, it is necessary to know both f_a and n_e.

Supposing that $n_e = 3/2$ and $n_w = 4/3$,

$$\frac{1}{f_w} = \left[1 - \frac{3/2}{4/3}\frac{(1/3)}{(1/2)}\right] \cdot \frac{1}{f_a} = \left[1 - 3/4\right] \cdot \frac{1}{f_a} = \frac{1}{4 f_a}$$

So $f_w = 4 f_a$.

2. Focal length is $f = -30$ cm.
Object distance, $s_o = 10$ cm.



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = \left(\frac{1}{f} - \frac{1}{s_o}\right)^{-1} = \left(-\frac{1}{30\text{ cm}} - \frac{1}{10\text{ cm}}\right)^{-1}$$

$$= -7.5 \text{ cm}$$

Magnification is $m_T = -\frac{s_i}{s_o} = \frac{7.5\text{ cm}}{10\text{ cm}} = 0.75$

The image is virtual.
It is located 7.5 cm from the lens on the same side as the object.
The image is not inverted.
The height of the image is $(6\text{ cm})(0.75) = 4.5$ cm

3. The first surface forms a virtual image at a position s_i' given by

$$\frac{1}{s_o} + \frac{n}{s_i'} = \frac{n-1}{R}$$

$$\Rightarrow s_i' = n\left(\frac{n-1}{R} - \frac{1}{s_o}\right)^{-1}$$

$$= (1.5)\left(\frac{(1.5-1)}{(10\,cm)} - \frac{1}{120\,cm}\right)^{-1}$$

$$= 36\ cm.$$

This intermediate image is treated as the object for the second surface, where $s_o' = 20\,cm - 36\,cm = -16\,cm$.

The final image is then formed at a position s_i given by

$$\frac{n}{s_o'} + \frac{1}{s_i} = \frac{1-n}{-R}$$

$$s_i = \left(\frac{n-1}{R} - \frac{n}{s_o'}\right)^{-1}$$

$$= \left(\frac{(1.5-1)}{(10\,cm)} + \frac{1.5}{16\,cm}\right)^{-1}$$

$$= 6.96\,cm.$$

The problem can also be treated as a thick lens with focal length

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

$$= (n-1)\left[\frac{2}{R} - \frac{(n-1)}{n}\frac{d}{R^2}\right]$$

Since $R_2 = -R_1 = -R$.

Thus, $\frac{1}{f} = \frac{1}{15\,\text{cm}}$ so $f = 15\,\text{cm}$.

However, this focal length is measured from the principal planes, not the surface of the lens.

The first principal plane is located at

$$h_1 = \frac{-f(n-1)d}{n(-R)} = 10\ \text{cm}$$

and the second is located at

$$h_2 = -\frac{f(n-1)d}{nR} = -10\ \text{cm}.$$

In this case the object distance is

$s_o = 120\ \text{cm} + 10\ \text{cm} = 130\ \text{cm}$ and the image distance is $s_i' = \left(\frac{1}{f} - \frac{1}{s_o}\right)^{-1} = \left(\frac{1}{15\text{cm}} - \frac{1}{130\text{cm}}\right)^{-1} = 16.96\ \text{cm}$

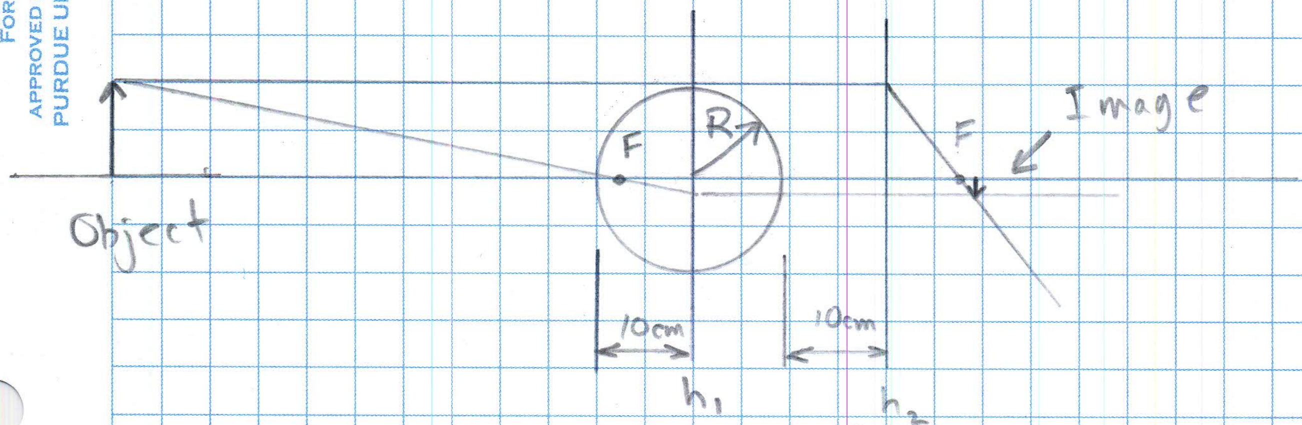
The distance from the vertex is then

$$s_i = s_i' + h_2 = 16.96\,\text{cm} - 10\,\text{cm} = 6.96\ \text{cm}.$$

The transverse magnification is

$$m_T = \frac{-s_i'}{s_o} = \frac{-16.96 \text{ cm}}{130 \text{ cm}} = -0.130 .$$

The ray diagram is as follows:



4. Gaussian lens formula:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

The parameters x_o and x_i in the Newtonian lens formula are

$$x_o = s_o - f$$
$$x_i = s_i - f$$

Hence, $$\frac{1}{x_o + f} + \frac{1}{x_i + f} = \frac{1}{f}$$

$$\frac{x_i + f + x_o + f}{(x_o + f)(x_i + f)} = \frac{1}{f}$$

$$\frac{(x_o + f)(x_i + f)}{x_o + x_i + 2f} = f$$

$$(x_o + f)(x_i + f) = f(x_o + x_i + 2f)$$

$$x_o x_i + (x_i + x_o) f + f^2 = (x_o + x_i) f + 2f^2$$

therefore, $$x_o x_i = f^2$$