Assignment # 8

1. When $\theta_i = \theta_B$ (Brewster's angle) then
$\theta_i + \theta_t = \pi/2$.

But $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's law)
and $\theta_t = \pi/2 - \theta_i$

So when $\theta_i = \theta_B$,

$$n_1 \sin\theta_B = n_2 \sin(\pi/2 - \theta_B)$$

But $\sin(\pi/2 - \theta_B) = \cos\theta_B$

so $n_1 \sin\theta_B = n_2 \cos\theta_B$

$$\frac{\sin\theta_B}{\cos\theta_B} = \frac{n_2}{n_1}$$

or $$\tan\theta_B = \frac{n_2}{n_1}$$

2. We expect that the specific rotation will be a linear function of the D-glucose fraction f_D. Thus, we can write

$$\alpha = a + b f_D .$$

When $f_D = 1$ (all D-glucose), $\alpha = +52°$

When $f_D = 0$ (all L-glucose), $\alpha = -52°$.

Thus, $a = -52°$ and $b = 2 \times 52° = 104°$.

So $$\alpha = (-52°) + (104°) f_D$$

3. First, we need to calculate the angle θ_t using Snell's law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where $n_1 = 1$, $n_2 = 1.5$ and $\theta_i = 30°$.
This gives

$$\sin\theta_t = \frac{\sin 30°}{1.5} = 0.333$$

so

$$\theta_t = 19.47°$$

(a) Using $R_\perp = \dfrac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$

and $R_\parallel = \dfrac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$

we have $R_\perp = \left(\dfrac{\sin(30° - 19.47°)}{\sin(30° + 19.47°)} \right)^2$

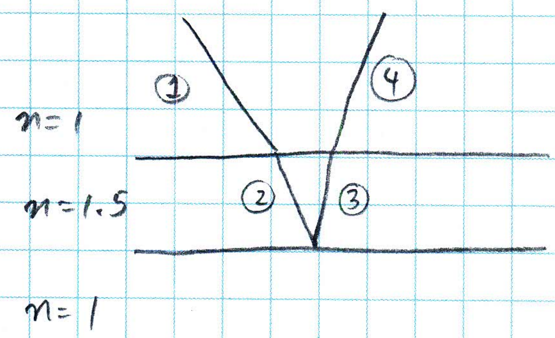
$$= \left(\frac{\sin 10.53°}{\sin 49.47°} \right)^2$$

$$= 0.0578$$

$$R_\parallel = \left(\frac{\tan 10.53°}{\tan 49.47°} \right)^2$$

$$= 0.0253$$

(b) To calculate the effective reflection coefficient when the light also reflects off the back surface, we will first need to calculate the intensity of light in each path shown below:



$$I_2 = I_1 T_{12}$$

$$I_3 = I_2 R_{21} = I_1 T_{12} R_{21}$$

$$I_4 = I_3 T_{21} = I_1 T_{12} R_{21} T_{21}$$

So we need to calculate T_{12}, T_{21} and R_{21} for each polarization state.

$$T_{12} = 1 - R_{12}$$

$$T_{\perp,12} = 1 - 0.0578 = 0.9422$$
$$T_{\parallel,12} = 1 - 0.0253 = 0.9747$$

$$R_{\perp,21} = \left(\frac{\sin(19.47° - 30°)}{\sin(19.47° + 30°)}\right)^2 = 0.0578$$

$$R_{\parallel,21} = \left(\frac{\tan(19.47° - 30°)}{\tan(19.47 + 30°)}\right)^2 = 0.0253$$

$$T_{\perp,21} = 1 - R_{\perp,21} = 0.9422$$
$$T_{\parallel,21} = 1 - R_{\parallel,21} = 0.9747$$

So $R'_\perp = 0.0578 + (0.9422)(0.0578)(0.9422)$

$= 0.0578 + 0.0513$

$= 0.109$

$R'_\parallel = 0.0253 + (0.9747)(0.0253)(0.9747)$

$= 0.0253 + 0.0240$

$= 0.0493$

In principle light could be reflected multiple times inside the slice of material so the effective reflection coefficients would be even larger than there.