Assignment #2.

1. The metal rod has length $L = 0.5\,m$, and a rectangular cross section of $A = 2\ mm^2$.

(a) If the rod is vertical and a mass of $M = 60\ kg$ is hung from one end and causes an extension of $\Delta z = 0.25\ mm$ what is Young's modulus of the material?

Young's modulus, Y, is defined by the relation,

$$\frac{F}{A} = \frac{Y}{L}\Delta z$$

Hence, $Y = \dfrac{FL}{A\Delta z}$ where $F = mg$.

$$Y = \frac{(9.81\ N/kg)(0.5\ m)(60\ kg)}{(2\times10^{-6}\ m^2)(0.25\times10^{-3}\ m)}$$

$$= .588.6 \times 10^9\ N/m^2.$$

(b) When the rod is clamped as shown in the Figure and a force, F, is applied in the y-direction the resulting deflection is

$$y = \frac{4L^3}{Yab^3} F .$$

If the force is removed, the motion of a mass m attached to the end will be determined by the differential equation

$$m\ddot{y} + k_y y = 0$$

where $k_y = \dfrac{Yab^3}{4L^3}$.

The frequency of oscillations in the y-direction will be

$$\omega_y = \sqrt{\frac{k_y}{m}} = \frac{1}{2}\sqrt{\frac{Yab^3}{mL^3}} .$$

Likewise the frequency of oscillations in the x-direction will be

$$\omega_x = \frac{1}{2}\sqrt{\frac{Ya^3 b}{mL^3}}$$

and their ratio will be

$$\frac{\omega_y}{\omega_x} = \sqrt{\frac{ab^3}{a^3 b}} = \sqrt{\frac{b^2}{a^2}} = \frac{b}{a} .$$

2. Damped, harmonic motion:

$$m\ddot{x} + b\dot{x} + kx = 0$$

We write this as

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

where $\gamma = b/m$ and $\omega_0 = \sqrt{k/m}$.

The frequency of oscillations $\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$ arises from the imaginary part of the roots of the characteristic polynomial:

$$(\alpha^2 + \gamma\alpha + \omega_0^2) = 0$$

$$\alpha = -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$$

We want to solve for γ and then for b when $\omega = \omega_0/2$:

$$\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = \frac{\omega_0}{2}$$

$$\Rightarrow \omega_0^2 - \frac{\gamma^2}{4} = \frac{\omega_0^2}{4}$$

$$\frac{\gamma^2}{4} = \frac{3\omega_0^2}{4}$$

$$\gamma = \omega_0\sqrt{3} = \sqrt{\frac{3k}{m}} = \frac{b}{m}$$

So $b = \sqrt{3km}$.