Assignment # 1.

1. Let $x(t) = \sum_{n=0}^{\infty} a_n t^n$. This can be rewritten in terms of even and odd powers of t:

$$x(t) = \sum_{n=0,2,4,\cdots} a_n t^n + \sum_{n=1,3,5,\cdots} b_n t^n$$

or

$$x(t) = \sum_{n=0}^{\infty} a_n t^{2n} + \sum_{n=1}^{\infty} b_n t^{2n-1}$$

Then
$$\frac{dx}{dt} = \sum_{n=0}^{\infty} 2n a_n t^{2n-1} + \sum_{n=1}^{\infty} b_n (2n-1) t^{2n-2}$$

$$\frac{d^2x}{dt^2} = \sum_{n=0}^{\infty} (2n)(2n-1) a_n t^{2n-2}$$
$$+ \sum_{n=1}^{\infty} b_n (2n-1)(2n-2) t^{2n-3}$$

If $x(t)$ satisfies $\frac{d^2x}{dt^2} + \omega^2 x = 0$

then for any n we must have

$$(2n)(2n-1) a_n + \omega^2 a_{n-1} = 0$$
$$(2n-1)(2n-2) b_n + \omega^2 b_{n-1} = 0$$

So
$$a_n = \frac{\omega^2 a_{n-1}}{(2n)(2n-1)}$$

$$b_n = \frac{\omega^2 b_{n-1}}{(2n-1)(2n-2)}$$

2. $x(t) = A\cos\omega t + B\sin\omega t$
is a solution to $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$.

This can be demonstrated:

$$\frac{dx}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$
$$= -\omega^2 x(t)$$

Thus, $\dfrac{d^2x}{dt^2} + \omega^2 x = (-\omega^2 x) + \omega^2 x = 0$.

If we write $x(t) = C\cos(\omega t + \alpha)$ then we need to find the relation between (A, B) and (C, α).

When $t = 0$ we have

$$x(t) = A = C\cos\alpha$$

When $t' = \dfrac{\pi}{2\omega}$ we have

$$x(t') = B = C\cos(\pi/2 + \alpha)$$
$$= -C\sin\alpha$$

Thus, $\tan\alpha = -B/A \Rightarrow \alpha = \tan^{-1}\left(\dfrac{-B}{A}\right)$

$$C = \sqrt{C^2\sin^2\alpha + C^2\cos^2\alpha} = \sqrt{A^2 + B^2}$$

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3. Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$

(a) $$\frac{e^{i\theta} + e^{-i\theta}}{2} = \frac{\cos\theta + i\sin\theta + \cos(-\theta) + i\sin(-\theta)}{2}$$

$$= \frac{\cos\theta + i\sin\theta + \cos\theta - i\sin\theta}{2}$$

$$= \frac{2\cos\theta}{2} = \cos\theta$$

(b) $$\frac{e^{i\theta} - e^{-i\theta}}{2i} = \frac{\cos\theta + i\sin\theta - \cos(-\theta) - i\sin(-\theta)}{2i}$$

$$= \frac{\cos\theta + i\sin\theta - \cos\theta + i\sin\theta}{2i}$$

$$= \frac{2i\sin\theta}{2i} = \sin\theta$$

(c) $e^{i\theta}e^{-i\theta} = 1$.

Therefore, $(\cos\theta + i\sin\theta)(\cos(-\theta) + i\sin(-\theta))$

$$= (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)$$

$$= \cos^2\theta - (i)^2\sin^2\theta$$

$$= \cos^2\theta + \sin^2\theta = 1.$$

(d) $$\cos^2\theta - \sin^2\theta = \left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)^2 - \left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^2$$

$$= \frac{e^{2i\theta} + 2 + e^{-2i\theta}}{4} - \frac{e^{2i\theta} - 2 + e^{-2i\theta}}{-4}$$

$$= \frac{e^{2i\theta} + e^{-2i\theta}}{4} + \frac{e^{2i\theta} + e^{-2i\theta}}{4} = \frac{e^{2i\theta} + e^{-2i\theta}}{2} = \cos 2\theta$$

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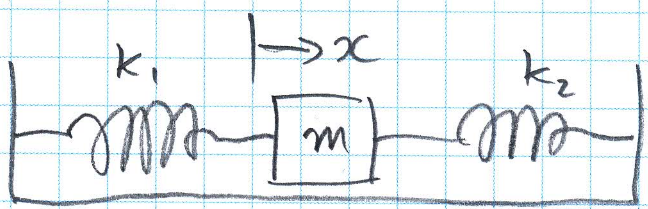
(e) $$2\sin\theta\cos\theta = 2\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)\left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)$$

$$= \frac{e^{2i\theta} - e^{-2i\theta}}{2i}$$

$$= \sin 2\theta$$

4.



When the mass is displaced from its equlilibrium position by a distance x, the force acting on it will be

$$F = -k_2 x - k_1 x .$$

Thus, the equation of motion is

$$m\frac{d^2x}{dt^2} = F = -k_1 x - k_2 x$$

which can be written in the standard form:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega^2 = \dfrac{k_1 + k_2}{m}$.

$$\Rightarrow \omega = \sqrt{\frac{k_1 + k_2}{m}} .$$