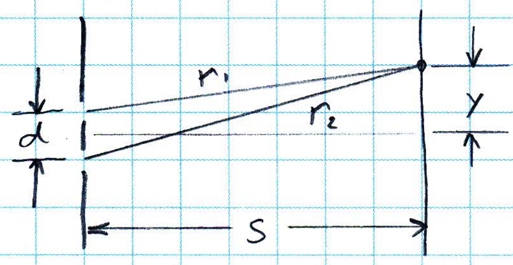
Assignment #10

1. Consider Young's double slit expriment without a glass sheet in front of one slit:



The number of wavelengths in r_1 is r_1/λ and the phase advance is kr_1. Likewise, the phase advance of the second path is kr_2 so the phase difference is $\delta = k(r_2 - r_1)$. The difference in path length is $\sim \frac{dy}{s}$ so the phase difference is $\delta \approx kdy/s$. The m^{th} maximum occurs at $\frac{kdy}{s} = 2\pi m$

So $y = \frac{2\pi m s}{kd} = \frac{\lambda m s}{d}$.

Now, if a glass sheet is placed in front of one of the slits, then the phase advance will be $k't + k(r-t)$ where $k' = nk$ is the wavenumber in the glass sheet of thickness t with index of refraction n. In this case,

$$\delta' = k't + k(r_2 - t) - kr_1$$

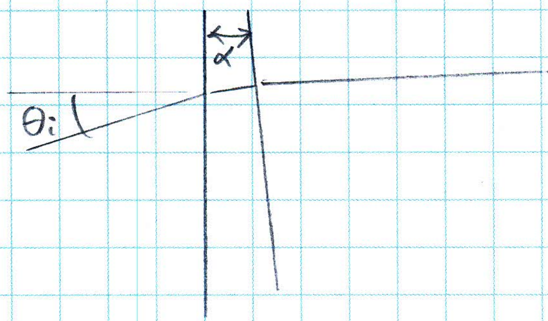
$$= kt(n-1) + \delta = \delta + kt(n-1)$$

$$= \frac{kdy'}{s} + kt(n-1) = 2\pi m$$

Thus, $y' = \frac{\lambda m s}{d} - \frac{st}{d}(n-1)$

Here, we assumed that $y/s \ll 1$.

2. Consider the change in angle as a ray passes through a prism with angle α:



At the first surface, the transmitted angle is given by Snell's law:

$$n'\theta = n\theta_t \Rightarrow \theta_t = \frac{n'\theta}{n}.$$

The ray then impinges on the second surface with an angle of incidence $\theta_i' = \theta_t - \alpha$ and is refracted to an angle

$$\theta_t' = \frac{n}{n'}\theta_i' = \frac{n}{n'}\left(\theta_t - \alpha\right) = \theta - \alpha\frac{n}{n'}$$

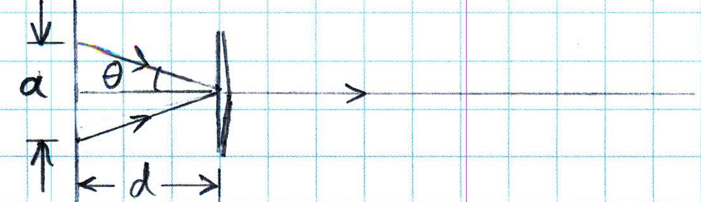
with respect to the second surface. This angle is then

$$\theta' = \theta_t' + \alpha = \theta - \frac{\alpha n}{n'} + \alpha$$
$$= \theta - \alpha\left(\frac{n-n'}{n'}\right).$$

When passing through the bottom part of the biprism, the angle of the refracted beam is

$$\theta' = \theta + \alpha\left(\frac{n-n'}{n'}\right).$$

The system acts like a double-slit experiment. The separation between the vertical sources can be calculated using $\theta' = 0$:



Thus, $\frac{1}{2}a = d\theta = d\alpha\left(\frac{n-n'}{n'}\right)$

So $a = 2d\alpha\left(\frac{n-n'}{n'}\right)$.

Once we ~~treat~~ treat the problem as a double-slit experiment we can calculate the path length difference at a point y on a screen a distance s from the source.

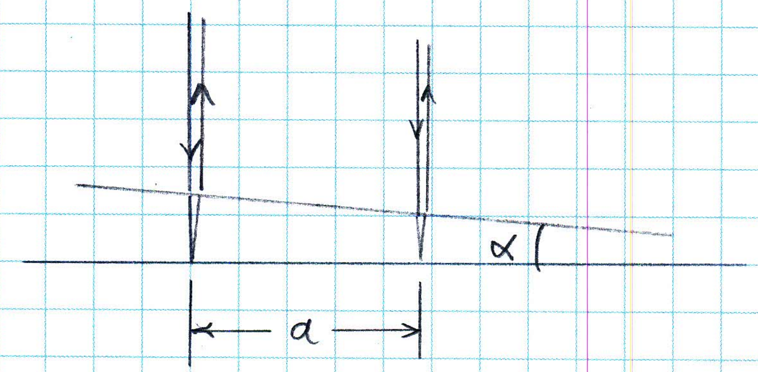
$$\delta = \frac{kay}{s} = \frac{2\pi n' a y}{\lambda_0 s} = 2\pi m$$

the fringe separation is then

$$\frac{n'a}{\lambda_0 s}\Delta y = 1 \quad \text{or} \quad \Delta y = \frac{\lambda s}{n'a} = \frac{\lambda_0 s}{2d\alpha(n-n')}$$

where λ_0 is the wavelength in free space.

3.



Separation between the fringes is a which corresponds to one wavelength difference in the optical path lengths.

The geometric path length difference is

$$\Delta d = 2 a \alpha$$

So $$2 a \alpha = \frac{\lambda_0}{n}$$

and $$\alpha = \frac{\lambda_0}{2 a n} = \frac{(500\,\text{nm})}{(2)(1/3\,\text{cm})(1.5)}$$

$$= \frac{500 \times 10^{-7}\,\text{cm}}{2 \times 1/3 \times 3/2\,\text{cm}} = 500 \times 10^{-7}$$

$$= 5 \times 10^{-5} \text{ radians.}$$

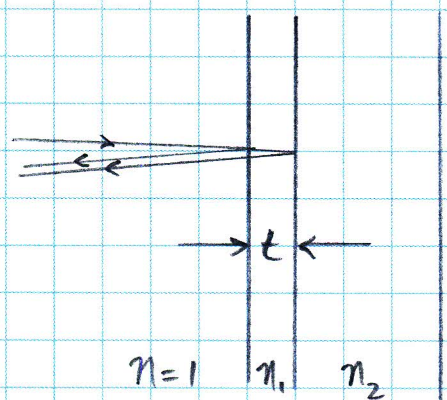
4. The optical path length of the chamber is $2nd$ and the number of fringes that shift as the optical path length is reduced to $2d$ will be

$$m = \frac{2(n-1)d}{\lambda_0}$$

$$= \frac{2(1.00029 - 1)(10\ \text{cm})}{600 \times 10^{-7}\ \text{cm}}$$

$$= 96.67 \approx 97.$$

5.



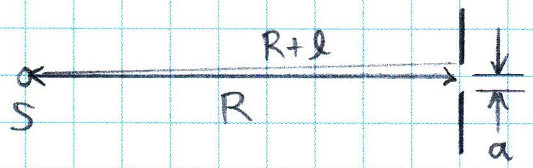
Since $n_1 > n$ and $n_2 > n_1$, there is a $180°$ phase shift at each interface. Thus, the phase difference is just due to the optical path length in the film: $2n_1t$.

If the reflected light is to be 1/2 a wavelength out of phase, then

$$2n_1t = \frac{\lambda_0}{2} \Rightarrow t = \frac{\lambda_0}{4n_1} = \frac{500\ \text{nm}}{4(1.30)}$$

$$= 96.2\,\text{nm}.$$

The cryolite film should be 96.2 nm thick.



$$(R+\ell)^2 = R^2 + a^2$$

$$\cancel{R^2} + 2R\ell + \ell^2 = \cancel{R^2} + a^2$$

Fraunhofer diffraction occurs when

$$\ell^2 + 2R\ell - a^2 = 0$$

$$\ell = -R + \sqrt{R^2 + a^2}$$

$$= -R + R\sqrt{1 + a^2/R^2}$$

$$= \frac{a^2}{2R} \ll \lambda$$

So $\dfrac{a^2}{2} \ll \lambda R$

Smallest R when $a = 1$ mm, $\ell = \dfrac{\lambda}{10}$, $\lambda = 500$ nm

$$\frac{\lambda}{10} = \frac{a^2}{2R}$$

$$R = \frac{10a^2}{2\lambda} = \frac{5(10^{-3}\,\text{m})^2}{500\times 10^{-9}\,\text{m}} = \frac{10^{-6}\,\text{m}}{10^{-7}} = 10\ \text{m}$$