Physics 422 - Spring 2013 - Final Exam, April 30$^{\text{th}}$

Answer all questions in the exam booklets provided.
There are 6 questions - please answer all of them.
Explain your reasoning clearly but concisely.
Clearly indicate which work is to be graded.
Each question is of equal weight.
You can use two pages of your own notes/formulas.

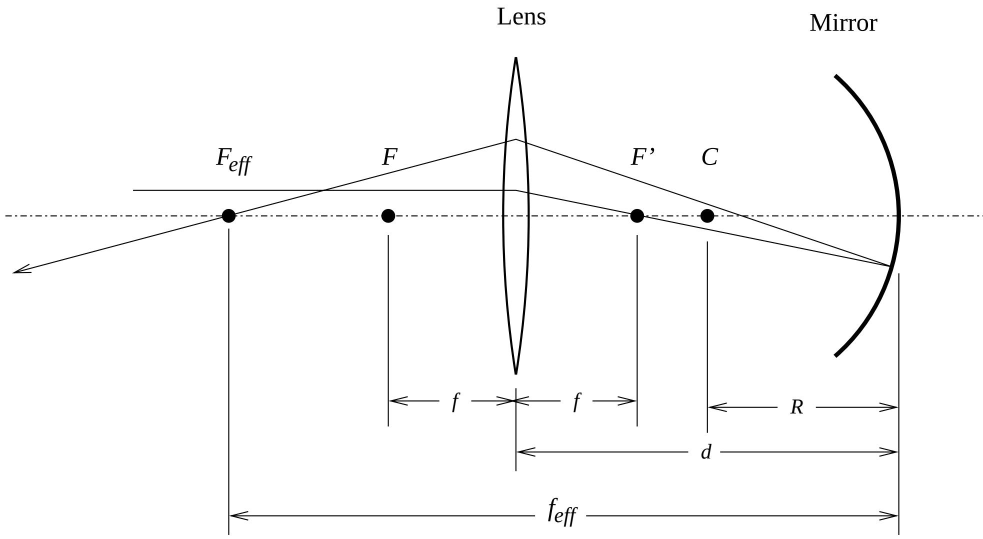
Here are some useful trigonometric identities:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\cos^2\theta - \sin^2\theta &= \cos(2\theta) \\
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin(2\theta) &= 2\sin\theta\cos\theta \\
\cos(2\theta) &= 2\cos^2\theta - 1
\end{aligned}$$

1. Show that the Stokes parameters for light that is linearly polarized along an axis that makes an angle θ with respect to the horizontal axis are

$$S = \begin{bmatrix} 1 \\ \cos 2\theta \\ \sin 2\theta \\ 0 \end{bmatrix}$$

2. Consider the following system consisting of a thin lens and a mirror:



For the following set of parameters:

$$\begin{aligned} R &= 30 \text{ cm} \\ d &= 60 \text{ cm} \\ f &= 20 \text{ cm} \end{aligned}$$

calculate the effective focal length, f_{eff} of the system, as shown on the diagram.

3. At the wavelength of the sodium D line ($\lambda = 590$ nm), the index of refraction of a type of glass varies as a function of temperature according to the formula

$$n(T) = n_0 + \alpha(T - 20)$$

where T is measured in degrees Celcius. If a sample of this type of glass that is 1 mm thick is placed in one arm of a Michelson interferometer and 50 fringes are observed to shift as the temperature of the glass is increased by 59° C, what is the value of the coefficient α?

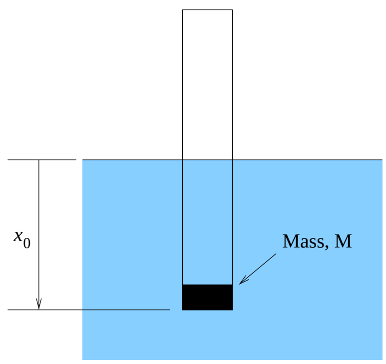
4. Answer one (and only one) of the following three questions:

(a) Describe the causes of two types of optical aberrations and techniques that can be employed to minimize them.

(b) Compare and contrast the conditions for Fraunhofer and Fresnel diffraction. Under what circumstances if Fraunhofer diffraction a special case of Fresnel diffraction?

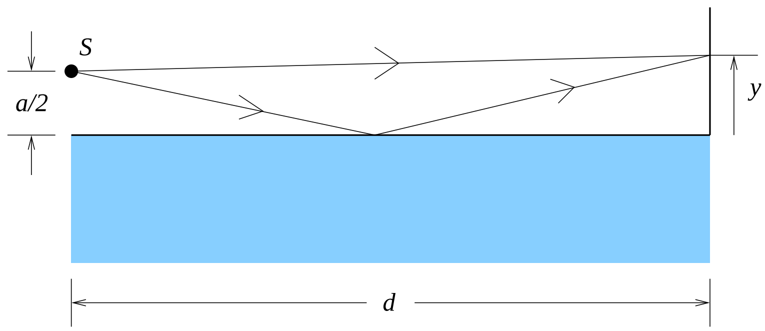
(c) Describe the geometries of three types of prisms, the paths that light rays make as they pass through the prisms, and the resulting changes in the orientation of images, if applicable.

5. A buoy floating in the ocean is made from a thin-walled cylindrical tube with radius a, that has a mass M at the bottom as shown:



(a) In the absence of any waves, calculate the equilibrium depth x_0 to which the bottom of the buoy will sink.
(b) If the buoy is displaced by an additional distance x from this equilibrium position, what will the net force acting on it be?
(c) If damping forces were very weak, what would be the frequency of oscillation, ω_0, of the buoy?
(d) The water exerts a damping force on the buoy that is of the form $F = -b\dot{x}$. If the amplitude of oscillations is reduced by half in a time T, what is the value of b/M?
(e) Assuming that $b/M \ll \omega_0^2$, if the buoy were driven by waves of amplitude A and frequency $\omega \approx \omega_0$, what would be the approximate amplitude of the buoy's steady state oscillations?

6. Light travelling in air directly from a source, S, and reflected from the surface of water ($n = 1.3$) interferes to create bright and dark fringes on a screen, located a distance d from the source as shown:



(a) At what height, y, are the bright fringes on the screen?

(b) If the light source and the screen are placed below the surface of the water, as shown, at what height, y', are the bright fringes seen on the screen?