Physics 422 - Spring 2013 - Assignment #7, Due March 1st

1. The radial wave equation in two dimensions is written:

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\omega^2}{v^2}\psi = 0.$$

Rotationally symmetric solutions to the wave equation in two dimensions which are finite at $r = 0$ are the Bessel functions, $J_0(z)$ where $z = kr$. Using the apprixmation for $J_0(z)$ when $z > 1$:

$$J_0(z) \sim \sqrt{\frac{2}{\pi}}\frac{\cos(z - \pi/4)}{\sqrt{z}}$$

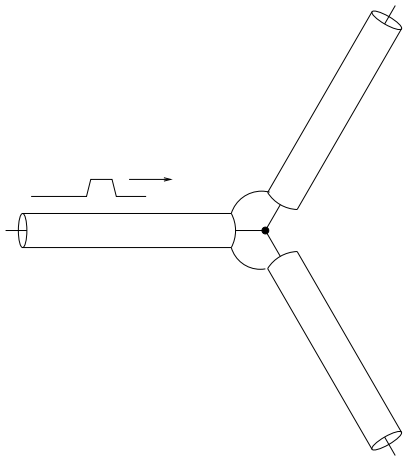
what would be the frequencies of the rotationally symmetric normal modes of oscillation for a circular drum with a radius R if the speed of wave propagation was v? Note that the drum imposes the boundary condition that $\psi(R, t) = 0$.

2. Consider a spherical balloon of radius R. Given that the speed of sound in air at a pressure p is given by

$$v = \sqrt{\gamma p/\rho}$$

where ρ is the density of air, what are the frequencies of the normal modes of the oscillations in the air contained in the balloon as a function of the pressure p? Clearly explain your reasoning.

3. Consider a transmission line with characteristic impedance Z which is connected to a pair of identical transmission lines as shown:



(a) The power carried by a pulse with current I is given by $P = ZI^2$. Given that the current would be split equally between the two transmission lines on the right, what is the effective impedance of this *pair* of transmission lines?
(b) If a pulse, incident from the left as shown, has an amplitude of V_i, what is the amplitude of the pulse that is reflected from the point at which the transmission lines are connected? Is the pulse inverted?
(c) What are the amplitudes of the transmitted pulses that propagate on each of the transmission lines on the right of the connection point?