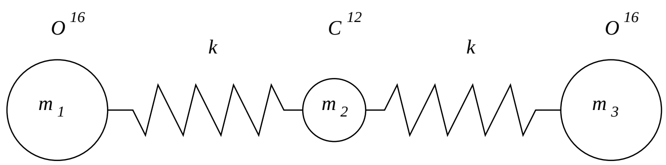
Physics 422 - Spring 2013 - Assignment #4, Due February 8st

1. (*French, 5-9*) The CO_2 molecule can be likened to a system made up of a central mass m_2 connected by equal springs of spring constant k to two masses m_1 and m_2 (with $m_1 = m_2$) as shown:

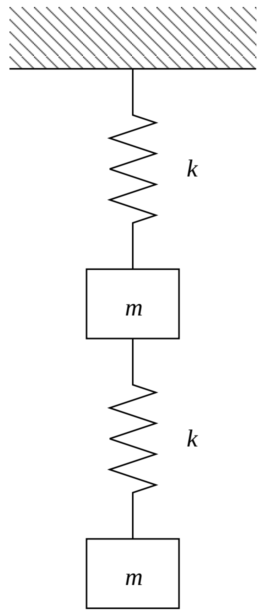


(a) Set up and solve the equations for the two normal modes in which the masses oscillate along the line joining their centers. The equation of motion for m_3 is

$$m_3 \frac{d^2 x_3}{dt^2} = -k(x_3 - x_2)$$

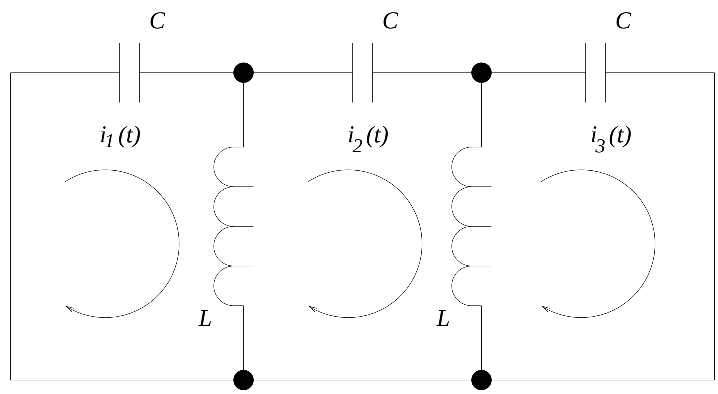
and there are similar equations for m_1 and m_2.
(b) Putting $m_1 = m_3 = 16$ units, and $m_2 = 12$ units, what would be the ratio of the frequencies of the two modes, assuming this classical description were applicable?

2. (*French, 5-10*) Two equal masses are connected as shown with two identical massless springs of spring constant k. Considering only motion in the vertical direction, show that the angular frequencies of the two normal modes are given by $\omega^2 = (3 \pm \sqrt{5})k/2m$ and hence that the ratio of the normal mode frequencies is $(\sqrt{5}+1)/(\sqrt{5}-1)$. Find the ratio of amplitudes of the two masses in each separate mode.



Note: You don't need to consider the gravitational forces acting on the masses because these are independent of the displacements.

3. Consider the following three-loop circuit in which all three capacitors have the same capacitance, C, and the two inductors have the same inductance, L.



(a) Write the set of three coupled, differential equations that describes the currents that flow in each loop.
(b) Show that solutions of the form $i(t) = Ie^{i\omega t}$ will satisfy the set of differential equations, provided ω satisfies the following matrix equation, in which $\omega_0 = \sqrt{1/LC}$:

$$\begin{pmatrix} \omega_0^2 - \omega^2 & \omega^2 & 0 \\ \omega^2 & \omega_0^2 - 2\omega^2 & \omega^2 \\ 0 & \omega^2 & \omega_0^2 - \omega^2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = 0$$

(c) Calculate the frequencies of the three normal modes of oscillation, expressing them in terms of the natural oscillation frequency $\omega_0 = \sqrt{1/LC}$.