Physics 422 - Spring 2013 - Assignment #3, Due February 1st

1. (*French, 4-4*) A block of mass m is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system:

(1) If the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.

(2) The viscous resistive force is equal to mg if the block moves with a certain known speed u.

(a) For this complete system, including both the spring and the damper, write the differential equation governing horizontal oscillations of the mass in terms of m, g, h, and u.

Answer the following questions for the special case that $u = 3\sqrt{gh}$:

(b) What is the angular frequency of the damped oscillations?

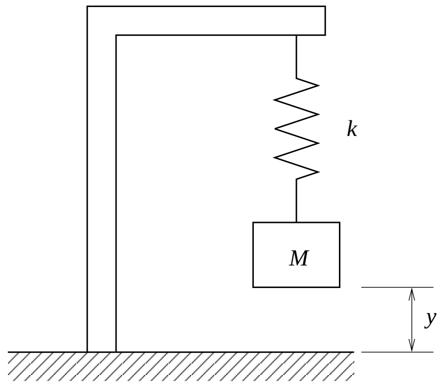
(c) After what time, expressed as a multiple of $\sqrt{h/g}$, is the *energy* down by a factor of $1/e$?

(d) What is the Q of this oscillator?

(e) This oscillator, initially in its rest position, is suddenly set in motion at $t = 0$ by an impulse which imparts a non-zero momentum in the x-direction. Find the value of the phase angle δ in the equation $x(t) = Ae^{-\gamma t/2}\cos(\omega t - \delta)$ that describes the subsequent motion, and sketch $x(t)$ *vs* t for the first few cycles.

(f) If the oscillator is driven with a force $mg\cos\omega t$, where $\omega = \sqrt{2g/h}$, what is the amplitude of the steady-state response?

2. (*French, 4-6*) Imagine a simple seismograph consisting of a mass M hung from a spring on a rigid framework attached to the earth, as shown:



The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but this is not an inertial reference frame if its surface is moving which would be the case in the event of an earthquake. Instead, if we define the position of the earth's surface to be η in an inertial reference frame, then the mass would have a position $z = \eta + y$ in this reference frame which would then satisfy Newton's second law: $F = M\ddot{z}$.

(a) Using y to denote the displacement of M relative to the earth and η to denote the displacement of the earth's surface itself, show tha the equation of motion is

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = -\frac{d^2\eta}{dt^2}$$

(b) Solve for $y(t)$ when the system undergoes steady-state oscillations if $\eta(t) = C\cos\omega t$.

(c) Sketch a graph of the amplitude A of the displacement y as a function of ω, supposing C is the same for all ω.

(d) A typical long-period seismometer has a period of about 30 seconds and a Q of about 2. As a result of a violent earthquake the earth's surface may oscillate with a period of about 20 minutes and with an amplitude such that the maximum acceleration is about 10^{-9} m/s^{-2}. How small a value of A must be observable if this is to be detected?