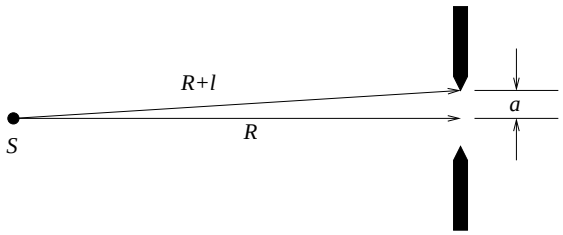
Physics 422 - Spring 2013 - Assignment #11, Due April 22th

1. (*Hecht, 10.1*) Consider the diagram below, in which a light source S is located a perpendicular distance R from a circular aperture of radius a. The conditions for Fraunhofer diffraction are that the difference, ℓ, between the paths at the periphery of an aperture and to its center is much less than the wavelength, λ.



(a) Show that this implies that Fraunhofer diffraction will occur when

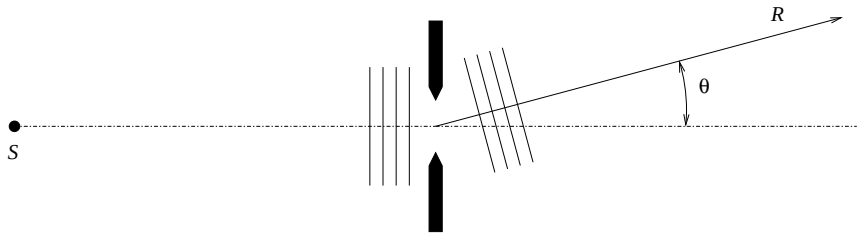
$$\lambda R \gg a^2/2$$

(b) What is the smallest satisfactory value of R if the hole has a radius of 1 mm, $\ell < \lambda/10$ and $\lambda = 500$ nm.

2. (*Hecht, 10.3*) When a plane wave of wavelength $\lambda = 2\pi/k$ impinges on a slit of width D, as shown below, the electric field at a distance R from the slit and at an angle θ is

$$E = \frac{D\mathcal{E}}{R}\frac{\sin\beta}{\beta}$$

where $\beta = \frac{1}{2}kD\sin\theta$.

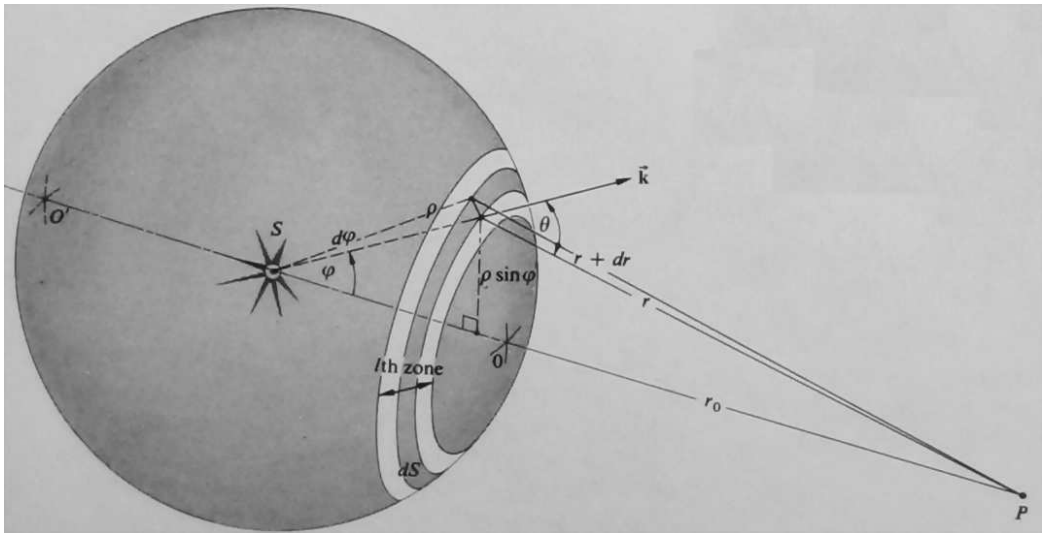


If the incident wave instead impinged on the slit at an angle θ_i, show that β in the expression above becomes $\beta' = \frac{1}{2}kD(\sin\theta - \sin\theta_i)$. Draw a diagram to explain your reasoning.

3. (*Hecht, 10.13*) Starting with the irradiance expression for a finite slit, show in the limit where the width of the slit approaches zero, that the intensity of the emitted light is the same in all directions.

4. (*Hecht, 10.33,39*) Light from a sodium lamp has two strong yellow components at 589.5923 nm and 588.9953 nm.
(a) How far apart in the first-order spectrum will these two lines be on a screen 1.00 m from a grating having 10,000 lines per centimeter?
(b) What is the total number of lines a grating must have in order to just separate the sodium doublet in the third order?

5. (*Hecht, 10.43*) Consider the following figure:



(a) Integrate the expression $dS = 2\pi\rho^2 \sin\varphi \, d\varphi$ over the ℓ^{th} zone to get the area of that zone,

$$A_\ell \;=\; \frac{\lambda\pi\rho}{\rho + r_0}\left[r_0 + \frac{(2\ell - 1)\lambda}{4}\right]$$

(b) Show that the mean distance to the ℓ^{th} zone is

$$r_\ell = r_0 + \frac{(2\ell - 1)\lambda}{4}$$

so that the ratio A_ℓ / r_ℓ is constant.