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Physics 34000 Laboratory
Scattering of Photons from Electrons: Compton Scattering

Objective: To measure the energy of high energy photons scattered from electrons in a metal as a function of scattering angle.

References:

1. A.H. Compton, Phys. Rev. **21**, 715 (1923)
   A.H. Compton, *The Spectrum of Scattered X-Rays*, Phys. Rev. **22**, 409 (1923).
2. A.C. Melissinos, *Experiments in Modern Physics*, Academic Press, New York, 1966, p. 252-65.
3. K. Krane, *Modern Physics*, 2nd Ed., Wiley and Sons, New York, 1996, p. 83-87.

Apparatus:

- Set of low-activity (1 μCi) γ-ray calibration sources (^{22}Na, ^{54}Mn, ^{57}Co, ^{60}Co, ^{109}Cd, ^{133}Ba, and ^{137}Cs)
- Photo-multiplier tube attached to a NaI(Tl) scintillator crystal with a lead shield
- Multi-channel analyzer PC peripheral interface
- High-activity (5 mCi) ^{137}Cs source encased in a lead shielding/collimator
- Metal rod for scattering γ-rays
- Movable carriage for changing observing angle

Introduction:
In 1923, Compton considered the problem of high energy photons (γ-rays) scattering from solids. Experimentally, he found that low energy (few MeV) monochromatic photons scattered by metals change their frequency and that the frequency change depends on the scattering angle. This proved to be problematic, since at that time, light scattering was understood in terms of diffraction in which the scattered (diffracted) wave does NOT change frequency. Compton's experiments and his theoretical analysis of them came to be known as Compton scattering. Historically, his experiments are important because they provided further compelling evidence that photons do behave as particles which obey conservation of momentum and energy laws. Compton was awarded the Nobel prize in 1927 for his seminal work.

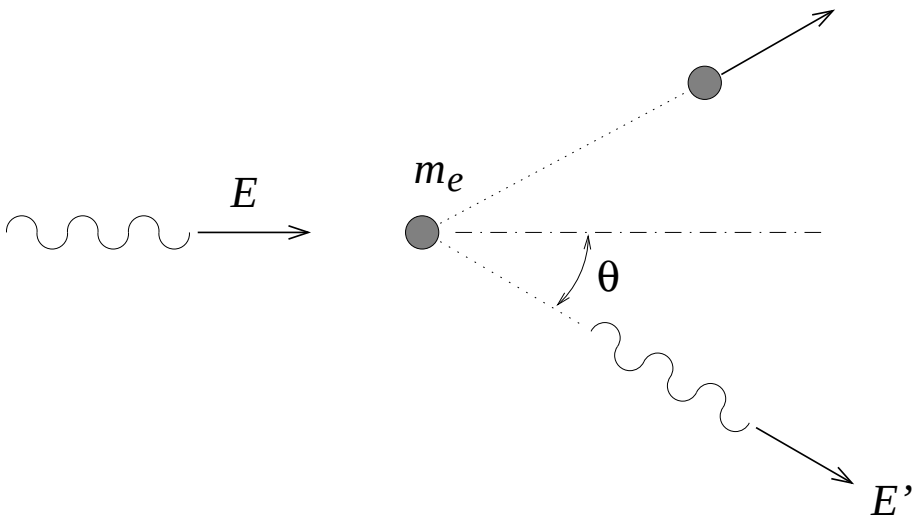

Figure 1: A schematic diagram showing the kinematic variables used to describe the scattering of an incident photon with energy E from an electron with mass m_e, initially at rest.

Compton's experiment can be understood by considering the interaction of the incident photons with the electrons that comprise a metal. If the quantized nature of electromagnetic radiation is taken into account (electromagnetic radiation consists of photons, each of which has the same energy, $E = h\nu$), and relativistic kinematics are used to describe the scattering process, the change in wavelength is understandable as a straight forward consequence of total energy and momentum conservation during a scattering process in which an incoming photon loses some of its energy to a an electron with mass m_e. The basic kinematic diagram illustrating this interaction is sketched in Figure 1.

For a beam of incident photons, each of which has the same energy, $E = h\nu$, there will be photons emerging at various angles, θ, with respect to the incident photon direction. The energy E' of a photon emerging at an angle θ can be calculated using relativistic kinematics and is described by the expression

$$E' = \frac{E}{1 + (E/m_e c^2)(1 - \cos\theta)}. \tag{1}$$

From Equation 1 it can be seen that in order to obtain a large Compton shift (*i.e.* a large value of $E - E'$), the incident photons should have an energy E that is comparable to the rest-energy of the electron, $m_e c^2 = 511$ keV. In this experiment, a collimated beam of 662 keV gamma rays from a ^{137}Cs source will be scattered by an aluminum rod.

Experimental Considerations

NaI(Tl) Crystal Scintillator

The energies of gamma rays from the decays of radioactive isotopes can be measured using an inorganic crystal scintillation detector. A crystal of sodium iodide, doped with a small admixture of thallium, is used as the active detector element. An incident photon can scatter from the electrons in the crystal which then deposit their energy in the crystal by ionizing other atoms in the crystal lattice. The electrons that are liberated in this way eventually recombine with the holes left in the lattice and emit photons with a range of wavelengths that peaks at about 400 nm, in the violet region of the visible spectrum. In this way, the number of photons produced is proportional to the energy of the incident photon. Because sodium iodide is a hygroscopic crystal, it must be sealed in an aluminum can to prevent it from absorbing moisture from the air which would ruin its optical properties.

Photo-multiplier Tubes (PMT)

A photo-multiplier tube is a vacuum tube that produces an electrical pulse that has an amplitude proportional to the amount of light that is incident on a thin, semi-transparent glass window. The inside surface of the window has a very thin coating of metal alkali metals, which have low work-functions, allowing an incident photon to eject an electron via the photoelectric effect. This surface is held at a large negative electric potential relative to other elements in the photo-multiplier tubes and the ejected electrons are accelerated away from the photo-cathode and can gain several hundred eV before they impact the first dynode. Dynodes are coated with a material such as beryllium copper oxide, that will emit several low-energy electrons when hit by an incident electron. Several dynode stages with increasing electric potentials allows the charge of the electron initially ejected from the photo-cathode to be multiplied by a factor as large as 10^5 or 10^6, producing an electric pulse at the output that has an amplitude large enough to be easily measured by relatively unsophisticated electronics.

Photo-multiplier tubes require a high voltage power supply to provide the accelerating potentials across the dynodes. Typical operating voltages are of order 1 kV but the circuit used to provide the voltages to the dynodes usually draws less than 1 mA of current. The gain of a photo-multiplier typically varies with the applied voltage according to $G \propto V^{\beta}$ where β can be as large as 5-6. Therefore, even small changes in the operating voltage can result in large changes in the gain. For this reason, precise photon energy measurements need a high voltage power supply that is very stable in time.

Although the output of a photo-multiplier tube is proportional to the amount of incident light, the power supply may not be able to deliver enough current to produce very large pulses from, for example, high energy photons incident on a NaI(Tl) crystal. In addition, the available current may be insufficient even for moderate pulses at very high rates. Thus, it is possible that the gain of a photo-multiplier tube is slightly non-linear, becoming slightly less than expected for large pulses.

Multi-channel Analyzer (MCA)
A multi-channel analyzer (MCA) detects electrical pulses at its input, measures the amplitude (or charge), and stores the resulting measurements in a histogram. The signal from the PMT is connected to the input MCA where it can be amplified by a pre-amplifier with a selectable gain. A discriminator triggers the electronics to measure the amplitude of a pulse when the signal from the pre-amplifier exceeds a specified threshold. This threshold can be set to $1 - 2\%$ full scale to ignore the large number of very small pulses due to electronic noise in the system. The amplitudes of triggered pulses are measured using a 10-bit analog-to-digital converter (ADC) and are stored in memory as a histogram.

The *dead-time* is the fraction of the time in which the MCA is measuring and analyzing pulses. If the dead-time is significantly larger than a few percent, then the probability that two photons could arrive at the same time becomes significant. This could degrade the energy resolution or bias the energy measurement. Therefore, keep the dead-time low by increasing the discriminator threshold or by reducing the intensity of the beam to which the PMT is exposed.

γ-Spectra of Calibration Sources
The relation between the energy of an incident photon and the multichannel analyzer channel number can be determined by measuring the positions of γ peaks with known energies from a range of isotopes. The spectrum from ^{22}Na is shown in Figure 2. In this example, an electron from an inner atomic shell is absorbed by the ^{22}Na nucleus, emitting a photon with an energy of 0.511 MeV. This almost always produces a ^{22}Ne nucleus in an excited state that subsequently decays to its ground state by emitting a second photon with an energy of 1.274 MeV. Although the photons are emitted in random directions, sometimes both are incident on the detector and the sum of their energies are measured. These three processes contribute to the three peaks observable in the ^{22}Na spectrum and all three peaks can be used to calibrate the relation between energy and channel number. Several other isotopes with well defined γ peaks can be used to provide other data for the calibration of the energy response of a scintillator+PMT+MCA system.

Radiation Safety
The calibration sources used in this lab have very low activity and do not present an exposure risk. The stronger, ^{137}Cs sources are contained in a lead shield and collimate the emitted photons so that they are directed away from an experimenter. Nevertheless, it is useful to use good source handling practices. In particular,

- Maximize the distance between you and a source. For example, use tongs to handle and position the calibration sources.
- Reduce the time exposed to a source of radiation. Do not spend an unnecessary amount of time when positioning the photo-multiplier tube in the collimated beam of the ^{137}Cs source.

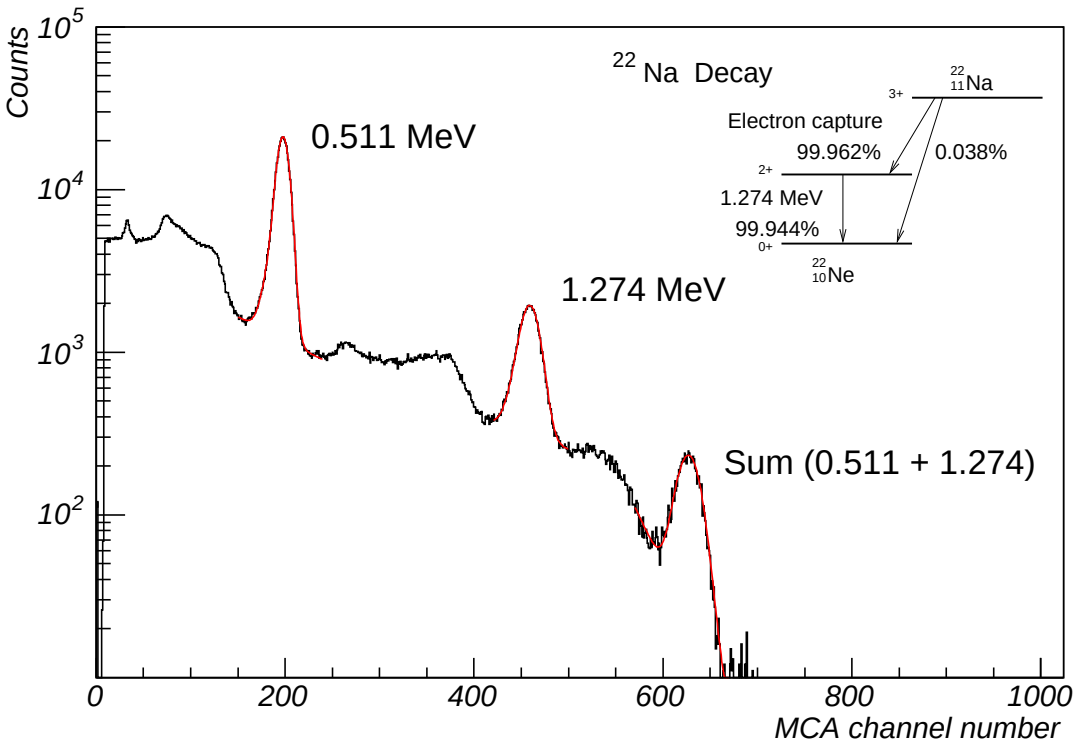

Figure 2: The decay scheme and observed energy spectrum for the decay of ^{22}Na.

- Use shielding to prevent exposure. When working with calibration sources, keep the lead shield in place on the ^{137}Cs source.
- Avoid unintentional ingestion of radioactive isotopes. Although all the sources used in this lab are sealed and cannot contaminate the surfaces in the lab, it is good laboratory practice to avoid ingestion of any contaminated material. Therefore, do not eat, drink, apply cosmetics, smoke, or chew tobacco in the lab. Also wash your hands after working in the laboratory.

Calibration Procedure

1. Dependence of PMT gain on voltage

   - Set the high voltage to approximately 950 volts and the internal pre-amplifier gain to 8.
   - Select a calibration source such as ^{60}Co or ^{22}Na, which have well-defined peaks in the energy range 1-2 MeV.
   - Verify that the energy resolution is good enough to clearly resolve the peaks. Adjust the high voltage and coarse gain setting if necessary such that a peak in the 1-2 MeV energy range occurs somewhere between channels 400 and 800.
   - Measure the centroid of the peak as the high voltage is varied over the range ± 50 volts.
   - Plot the measured position as a function of voltage on a log-log plot. Compare the relationship with $G = G_0(V/V_0)^\beta$ where G_0 is the gain at a reference voltage, V_0.
   - Calculate the percentage change in the gain that would result if the power supply output fluctuated by 0.1%.

2. Calibration of MCA channel number
   For the Compton scattering part of the lab you will study photons with energies less than 1 MeV. Therefore, the gain should be adjusted so that the photon peak from ^{137}Cs is located near channel 500. This is typically achieved with a high voltage of approximately **950 V**, and the pre-amplifier gain set to **16**.

   - Set the high voltage and pre-amplifier gain so that the ^{137}Cs peak is somewhere between channels 500 and 700.
   - Record the spectra of several calibration sources, recording their peak positions estimated using the peak finding analysis provided by the MCA interface software.

- With the high voltage and gain selected above, not all of the peaks will be within the range of channels analyzed by the MCA. Identify which sources have well-defined peaks in the range of channels that you can observe and identify the energies of these peaks.
- You can save a spectra to a thumb drive using 'Save-as' and selecting the 'TKA' file format. Each line of this file consist of the number of counts in each of the 1024 channels and is suitable for importing into Microsoft Excel.
- Look up the energies of the dominant peaks for the isotopes studied and plot a graph of channel number as a function of energy. Fit this curve to a straight line and a 2^{nd} order polynomial. Which functional form provides the best fit to the data?
- It may be convenient to also fit a function to energy vs channel number to facilitate the conversion of channel numbers into energy units in subsequent parts of the lab.

Study of Compton Scattering

1. Estimate the angular resolution of the photo-multiplier tube as shown in Figure 3. The uncertainty in the angle can be estimated using $\sigma_\theta \approx \Delta\theta/\sqrt{12}$.
2. Remove the lead cover from the ^{137}Cs source and position the PMT on the movable carriage at approximately 5°. Locate and measure the position of the ^{137}Cs peak and compare this with the results of the ^{137}Cs calibration source.
3. Place the aluminum scattering rod on the holder and repeat the measurement, recording sufficient data to obtain an accurate estimate of the peak position. Record the live-time over which data was accumulated, the angle and the measured centroid of the peak.
4. Repeat the previous step for an angle of 10°.
5. Return to the angle used in step 2 and measure the peak position again to determine if it has drifted for any reason.
6. Repeat steps 3 and 4 for angles up to 90°, always returning to the initial position of 5° to monitor changes in the PMT gain. Because the intensity of photons scattered at large angles decreases, you will have to accumulate data for longer periods at larger angles.

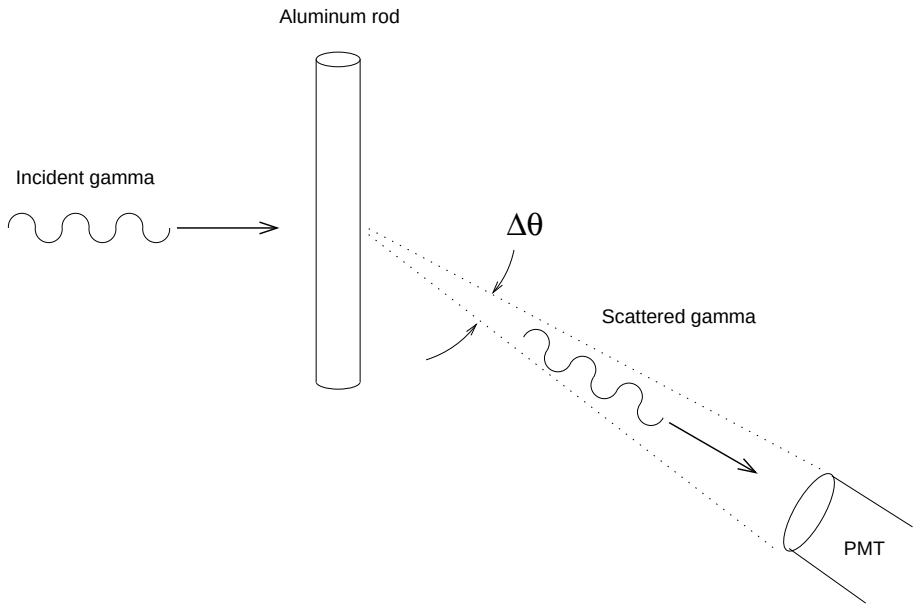

Figure 3: Method for estimating the uncertainty in the scattering angle.

Data Analysis

1. Using the fitted curve to the MCA channel number as a function of photon energy, convert the channel numbers of the measured peak positions to energies.

2. Measure the mean and RMS of the distribution of photon energies recorded with the PMT at 5°. The RMS of this distribution can be used to estimate the uncertainty on any given measurement since it reflects the size of any uncontrolled fluctuations in gain.

3. Tabulate θ, peak position, peak energy (E') and their uncertainties for all angles at which data was taken. Include this table in your report. Make sure you explain and shown sample calculations for the uncertainties in these quantities.

4. Plot $1/E'$ as a function of $1 - \cos\theta$. State what can be concluded from this plot. Be sure to include both horizontal and vertical error bars in your plot.

5. Perform a least-squares fit of a straight line to the data plotted above. From the slope, determine a value of the rest energy of the electron, $m_e c^2$. Be sure to show a detailed calculation of the uncertainty in this quantity in your lab book. Discuss how your value compares with the accepted value.