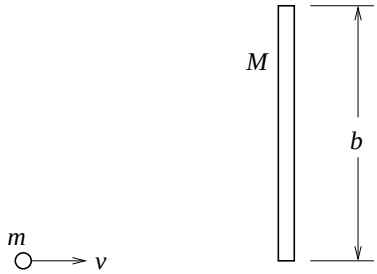
Physics 310 - mid-term exam - November 16 2004

Instructions: Answer all questions in the exam booklets provided. You are not permitted to use reference materials, including the text, lecture notes, past assignments or formula sheets, nor will there be any need to use a calculator. You have 2 hours to complete the exam.

1. Before driving to Purdue, a physics professor leaves his Classical Mechanics text book on the roof of his 1999 Toyota Corolla. If the mass of the text book is m and the coefficient of static friction between the roof and the text book is μ, what is the maximum acceleration that will be possible before the book slips off?

2. A bug with mass m is located at a distance b from the center of a turn-table that begins to rotate with constant angular acceleration, α. If the coefficient of static friction between the bug and the surface of the turn-table is μ, calculate the time at which the bug slips.
Hint: $\vec{a} = \vec{a}' + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')$.

3. A ball of putty with mass m moves with speed v and hits on end of a thin rod of length b and mass M as shown:



(a) Calculate the moment of inertia of the rod about an axis perpendicular to the rod at a distance x from its center of mass.

(b) If the collision is completely inelastic and the ball of putty sticks to the end of the rod, calculate the new position of the center of mass of the system.

(c) Calculate the final moment of inertia of the system about the new center of mass.

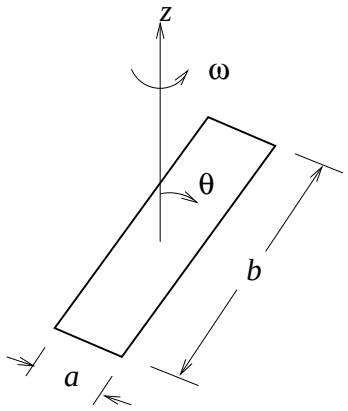
(c) Calculate the final velocity of the center of mass of the system.

(d) What is the initial angular momentum of the system? What is the final angular momentum of the system? Calculate the final angular velocity about the center of mass.

4. Consider a thin rectangular plate with width a and length b.

(a) If the plate lies in the x-y plane, calculate the elements of the moment of inertia tensor when the plate spins about one of the cartesian axes though its center of mass. Are the cartesian axes also the principal axes?

(b) The plate is oriented at an angle θ with respect to the z-axis and it spins about the z-axis with constant angular velocity ω as shown:



Calculate the magnitude and direction of the torque that must be present to keep the plate spinning in this orientation. Use Euler's equations:

$$\begin{aligned} N_1 &= I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) \\ N_2 &= I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3) \\ N_3 &= I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) \end{aligned}$$