Physics 310 - mid-term exam - October 5, 2004

Instructions: Answer all questions in the exam booklets provided. You are not permitted to use reference materials, including the text, lecture notes, past assignments or formula sheets, nor will there be any need to use a calculator. You have 2 hours to complete the exam.

1. A bead slides on a wire bent into the form of a helix. The motion of the bead is given in cylindrical coordinates by

$$\begin{aligned} r(t) &= b = \text{constant} \\ \phi(t) &= \omega t \\ z(t) &= ct \end{aligned}$$

   Find the velocity and acceleration vectors as functions of time.

2. A ball of mass m falls under the influence of gravity in a very sticky material that slows the ball down with a force given by $-c_3 v^3$ where v is the velocity of the ball. Calculate the terminal velocity of the ball as it falls in this medium.

3. The force acting on a particle of mass m is given by

$$F = kvx$$

   in which x is the position of the particle, v is its velocity and k is a positive constant. If the particle passes through the origin with speed v_0 in the $+x$ direction at $t = 0$,
   (a) Calculate v as a function of x.
   (b) Calculate x as a function of t. Recall that

$$\int \frac{dy}{y^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{y}{a}$$

4. Under what conditions will the force

$$\begin{aligned} F_x &= ayz \\ F_y &= bxz \\ F_z &= cxy \end{aligned}$$

   be conservative?

5. The force on an electron that moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

(a) For the case where $\vec{B} = \hat{k}B$, write the three components of the equation $\vec{F} = md^2\vec{x}/dt^2$.
(b) Solve the equations to obtain $\vec{x}(t)$.
(c) What is the relationship between the component of the electron's momentum in the x-y plane and the radius of curvature of its trajectory?

6. A particle of mass m is described by a driven, damped harmonic oscillator by the differential equation

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

Give a qualitative description of the motion of the particle and compare the different behaviors when
(a) $\omega \ll \sqrt{k/m}$
(b) $\omega \approx \sqrt{k/m}$
(c) $\omega \gg \sqrt{k/m}$