Physics 310 - final exam - December 14, 2004

Instructions: Answer all questions in the exam booklets provided. You are not permitted to use reference materials, including the text, lecture notes, past assignments or formula sheets, nor will there be any need to use a calculator. You have 2 hours to complete the exam.

Formulas:

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} + \lambda(t)\frac{\partial f}{\partial q_i} = 0$$

1. For reasons that were not immediately obvious to the investigating officer, a physics professor parks his 1999 Toyota Corolla at the top of the Great Lakes Chemical Company World Headquarters building as shown:

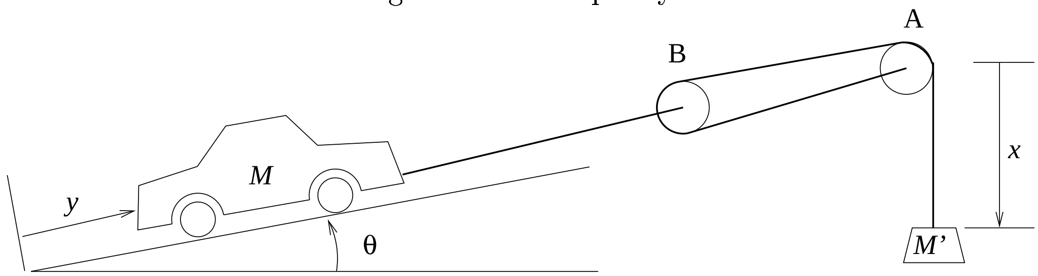
When he gets out of the car to explain the situation, he leaves it in gear and it begins to roll down the the slope, which makes an angle θ with respect to the horizontal. The car has a mass M and the engine of the car has a moment of inertia I, and is connected to the wheels, which have diameter d, with a gear ratio k. That is, the engine turns over k times for each revolution of the wheels.

(a) Use the position of the car on the slope and the angle of the internal engine parts as generalized coordinates to write down the kinetic and potential energies of the car.

(b) Write down an equation of constraint that relates the position of the car and the angle of the internal engine parts.

(c) Write down the Lagrangian for the system and find the acceleration of the car down the slope, by *either* elliminating one of the generalized coordinates using the constraint, or using Lagrange multipliers.

2. After driving his 1999 Toyota Corolla into the Wabash river, a physics professor pulls it out using a mass M' connected to an arrangement of two pulleys as shown:



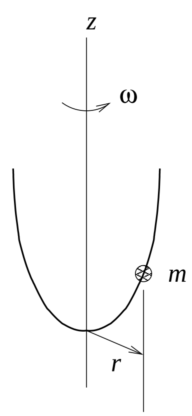
In this figure, the bank of the river is at an angle θ with respect to the horizontal. Pulley "A" is fixed in place and does not move, while pulley "B" is free to move. The car has mass M which is much greater than the mass of either pulley.

(a) Using the distance, y that the car has moved up the bank and the distance, x, that the cable has been pulled, write expressions for the kinetic and potential energy of the system.

(b) Write down an equation of constraint that relates x and y.

(c) Using Lagrange multipliers, determine the acceleration of the car and the tension in the cable from Lagrange's equations.

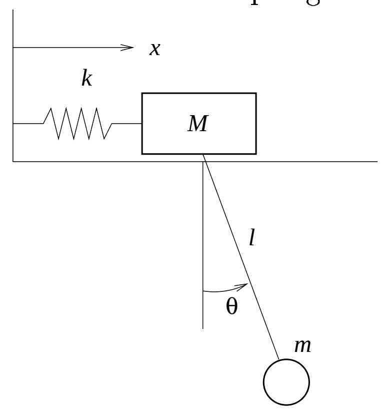
3. A piece of wire is bent into the shape of a parabola described by the equation $y = ar^3$ and it rotates about its axis of symmetry with a constant angular velocity, ω. A small bead of mass m slides on the wire.



(a) Use Lagrange's equations to derive the equations of motion of the bead, with its position parameterized using the variable r. Which values of r yield a state of stable equilibrium?

(b) Calculate the frequency of small oscillations about a point of stable equilibrium.

4. A mass m hangs from a string of length l that is attached to a block of mass M that slides on a frictionless surface. The block is attached to a spring with spring constant k as shown:



Find the frequencies of the normal modes of oscillation of the system.