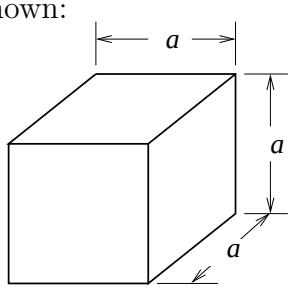
Physics 310 - Assignment #5 - Due Tuesday, November 30^{rd}

1. Consider a cube of mass m as shown:



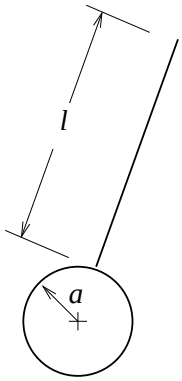
(a) The cube is forced to rotate with angular velocity $\vec{\omega} = \omega\hat{k}$ about its center of mass. Calculate the magnitude and direction of its angular momentum.

(b) In what direction are the principal axes when the cube rotates about its center of mass?

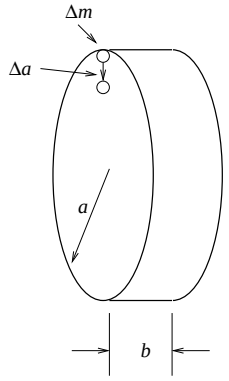
(c) Calculate the moment of inertia tensor when the cube is constrained to rotate about an axis that passes through one of its corners.

(d) Determine the directions of the three principal axes that pass through one corner of the cube and the principal moments about each principal axis.

2. Calculate the period of a pendulum made out of a sphere of radius a and mass m which hangs at the end of a thin string of length l, as shown:

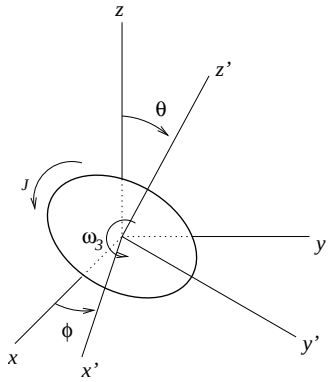


3. The wheel of my 1999 Toyota Corolla can be described approximately as a uniform disk of radius a and thickness b. In September, I hit a *tremendous* pot hole on Interstate 80-94 and bent the wheel so that a small amount of mass Δm located at the outer edge of the weel was moved towards the center by an amount Δa as shown:



(a) Calculate the moment of inertia tensor for the wheel when it rotates about what used to be the center of mass.
(b) If the wheel rotates with angular velocity ω, calculate the angle between the axis of rotation and the angular momentum vector.

4. A loop of wire of mass m and radius a carries an electric current J, and rotates with angular velocity ω in the orientation shown:



(a) If the loop is in a uniform magnetic field, $\vec{B} = B\hat{k}$, calculate the torque that it exerts on the loop about the x' axis. The force on an element current is given by

$$d\vec{F} = \frac{J}{c} d\vec{u} \times \vec{B} \tag{1}$$

where $d\vec{u}$ is an element of the loop in the direction of the current flow.

(b) Write down Euler's equations of motion for the rotating ring.

(c) Solve Euler's equations for the components of the angular velocity along the x', y' and z' axes.

(d) Calculate the component of the angular velocity along the z-axis. Part of this is due to the rotation of the ring about the z'-axis. Calculate the magnitude of the other part, which represents the speed at which the ring precesses about the z-axis.