Physics 310 - Assignment #3 - Due October 14^{th}

1. (a) Show that any force that is of the form

$$\vec{F}(\vec{r}) = f(r)\hat{r}$$

is conservative using the Cartesian representation, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$.

(b) Next, show that it is conservative using the representation of $\nabla \times \vec{Q}$ in spherical coordinates:

$$\nabla \times \vec{Q} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ Q_r & rQ_\theta & r\sin\theta\, Q_\phi \end{vmatrix} \frac{1}{r^2 \sin\theta}.$$

(c) The gradient in spherical coordinates is

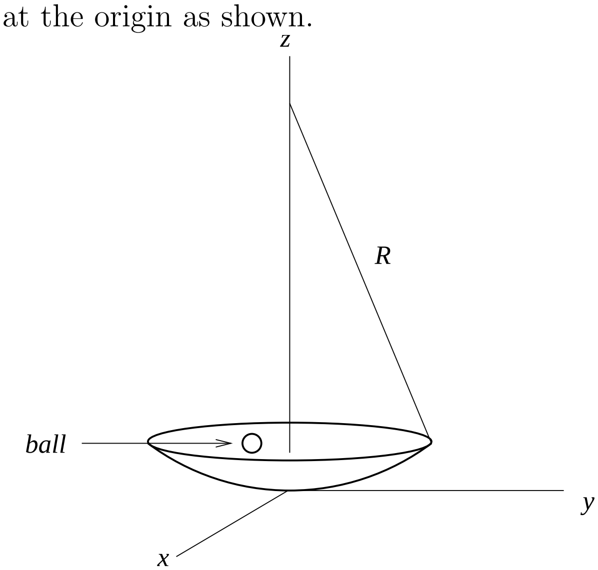
$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial f}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

Propose an expression for $V(r)$ in spherical coordinates that will satisfy $\vec{F}(\vec{r}) = -\nabla V(\vec{r})$.

2. *(Fowles and Cassiday, 4.17)* An electron moves in a force field due to a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ that is at right angles to $\vec{E}$. Let $\vec{E} = E\hat{j}$ and $\vec{B} = B\hat{k}$. Take the initial position of the electron at the origin with initial velocity $\vec{v}_0 = v_0\hat{i}$ in the x-direction. Find the resulting motion of the particle. Show that the path of motion is of the form:

$$\begin{aligned} x(t) &= a\sin\omega t + bt \\ y(t) &= a(1 - \cos\omega t) \\ z(t) &= 0 \end{aligned}$$

3. A ball of mass m rolls in a spherically shaped bowl with a radius of curvature, R, with its lowest point located at the origin as shown.



(a) Calculate the potential energy of the ball as a function of x and y.
(b) Calculate the components of the force that acts on the ball.
(c) For small displacements, $r = \sqrt{x^2 + y^2} \ll R$ from the origin, expand the potential as a Taylor series in r, keeping terms up to order r^2.
(d) What is the period of small oscillations in the x and y directions?

4. *(Fowles and Cassiday, 4.19)* A bead slides on a smooth rigid wire bent into the form of a circular loop of radius b. If the plane of the loop is veritcal, and if the bead starts from rest at a point that is level with the center of the loop, find the speed of the bead at the bottom and the reaction of the wire on the bead at that point.

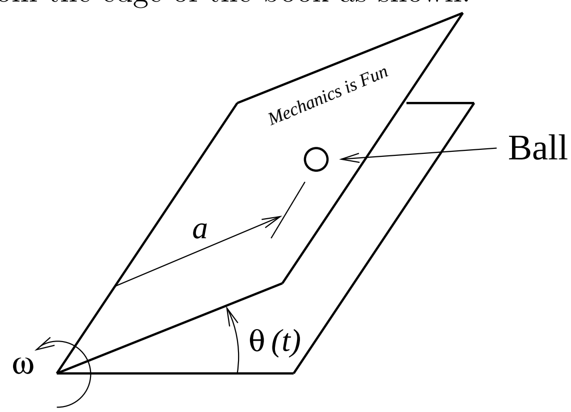
5. My 1999 Corolla has a drop of oil hanging underneath the engine. As I drive down interstate I-65, trying to arrive at Purdue in time to teach Physics 310, I drive over a dip in the road which makes my car travels in a path that is of the form:

$$\begin{aligned} x(t) &= vt \\ z(t) &= -h\exp(-x^2(t)/2\sigma^2) \end{aligned}$$

(a) Calculate the acceleration of my car as a function of time.
(b) Calculate the time at which the acceleration is a maximum.
(c) What is the minimum speed required to dislodge the oil drop if it takes twice the force of gravity to pull it off the engine?

6. A large mass, M, is attached to a spring with spring constant k and undergoes oscillations with a maximum amplitude of A. Sitting on top of this big mass is a smaller mass, m, attached to a spring with spring constant κ. What is the amplitude of oscillations of the small mass, measured in the non-inertial reference frame of the large mass? Assume that $m \ll M$, so that the motion of the small mass does not influence the motion of the large mass.

7. A ball of mass m sits on the cover of your closed Physics 310 text book. Suppose you open the book so that its cover moves with angular speed ω in such a way that the ball remains a constant distance a from the edge of the book as shown:



(a) Regarding the cover of the book as a rotating non-inertial reference frame, derive an equation relating ω to θ that will keep the ball stationary with respect to the cover of the book.
(b) Show that this gives a differential equation that, in principle, can be solved to give $\theta(t)$.
(c) Try to solve it... I may have to give a hint for the integral.